Teaching Portfolio

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October 30, 2022

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1 Introduction

This document describes my teaching and activities related to teaching while a graduate student at Indiana University, and a Visiting Assistant Professor at Williams College. In these sections you will find:

2. Teaching Statement. My teaching statement concerns my philosophy, style, and goals to grow as a teacher.

3. Documentation of Teaching. This section includes brief descriptions of the classes which I have taught, primarily concerning content and format, as well as my responsibilities.

4. Mentorship and Professional Development. In this section I describe some of the training I have received as a teacher, and how I have assisted with that training. I also describe my role in upper level undergraduate reading courses in math, peer mentoring, and teaching awards I have received.

Appendices. This section includes samples of materials I created for classes, including assignments (with samples of student work), class notes, organizational materials, and syllabuses.

2 Teaching Statement: Math is Hard

Theorem 1. *Math is hard.*

By this I mean that parts of math are hard. Some of my students don't know this yet, but some do. Some have even internalized Theorem 1 in ways that are not always healthy.

Theorem 2. *Math is worthwhile.*

Again, what I mean is that parts of math are worthwhile, because of utility or beauty. And again, some of my students know this, but some don't. I consider it a big part of my job to find proofs of Theorem 2 that are accessible to broad audiences. The strategies employed in that endeavor are informed heavily by the more general Theorem 3.

Theorem 3. *For any person, at any time, parts of math are hard, parts of math are worthwhile, and some parts are both, but which parts these are depends on the person and time.*

For example, much of what I once found hard and dubious now seems clear and magnificent. What is difficult before breakfast becomes easy after coffee. What feels worthwhile for a year becomes pointless after a moment of clarity.[1] A question asked in idle curiosity may pose challenging, and grow more compelling, until eventually an answer simply must be found.[2]

These theorems loom over any endeavor in math, and they make unexpected classroom visits. I prepare for such visitations, both irksome and wonderful, by meeting students where they are and focusing on their successes rather than my own expectations. I emphasize collaboration, communication skills, and creativity, because I believe these are relevant at any stage in the process of mathematics. And I strive to incorporate these ideals into my teaching habits as well as actionable course policies.

My students are collaborators in the learning process, which helps me recognize their needs. My lecturing style is very conversational, and my teaching style emphasizes collaboration and communication. In a recent class, I introduced the notion of a set partition, and asked students how many set partitions they could find of $\{1, 2, 3\}$. This was a vicious thing to do at 8:30 am, but the students got to discuss the problem with their peers which helped. There are five such partitions, but when I asked the students how many they found, the first student said 8.[1] The first example he gave, $\{\{1\}\}$, was not quite right as it did not account for 2 or 3, but once this had been explained several students were able to suggest the five correct partitions. The first student asked whether there is a formula that solves such a problem; the answer to that question is complicated, but the class had seen recurrence relations before and were prepared for a partial answer. Some students stuck around after class to find out more.[2] Organic discussions like these are slow, but I believe they are worthwhile. The class only solved the one problem during which time I could instead have presented several, but they also all engaged with the problem, and talked about it, and wanted to know more. And I learned how the students were doing, and learned a non-example to give the next time I teach this topic.

[1] An example of Theorem 1.
[2] An example of Theorem 2.

I take advantage of the space in a smaller classroom to use more involved groupwork. When teaching about Euler circuits, I start by challenging students to form groups and solve the classic Bridges of Königsberg problem, presenting a map of Königsberg and a corresponding graph. Many of the teams discover for themselves a reason why the problem is impossible. By the time I define the terms walk, Euler path, and Eulerian graph the students already have strong intuition for what those things are, and I am simply providing names for their ideas. When I present a characterization of Eulerian graphs, I am affirming or refining their own theories. The students remember these specific ideas better, but they have also gained experience working on problems without an obvious approach.

I write assignments with this kind of high-level engagement in mind, designing them as teaching tools, rather than just evaluation tools, so that classwork and homework are part of a continuous learning process. That process starts with students practicing core skills in the classroom with my support, includes plenty of direct applications of those skills for students to try on their own, and continues with students tackling problems requiring creative solutions. For example, when I teach voting theory, I start by teaching students about the rules of various voting systems used in the world today. After students learn to determine the winners of elections according to those rules, I ask them to design their own voting system with the goal of making it as democratic as possible. Although one of my first groups insisted that the "hunger games" was their final answer, most others evaluate which principles of established voting systems they like in order to adopt the best and reject the worst aspects of the systems they have studied. They have to think about voting systems in terms of their underlying philosophy, rather than treating them as number-pushing algorithms. The philosophical perspective clarifies why we study each system, and is also much easier to remember.

I teach with attention to the diverse needs of students, employing the principle of universal design when possible. I make sure that class discussions are accessible to students with weaker math backgrounds, while creating rubrics that do not needlessly emphasize their weaknesses (usually algebra). I make materials available to everyone, even if only one student requested the material. I advertise campus resources in class, like tutoring centers, but also counseling centers, rather than only sharing information with students who ask for it. I also recognize that students have complex lives that don't always stay outside of the classroom. My students know that my late homework policies are flexible, no questions asked, and they know it's because I care that they complete assignments. And I find that when I indulge students who need extra time, even when the root of the problem is time management, they tend to do better in the future.

The value of flexibility was the biggest lesson I learned about teaching during the first years of the pandemic. I taught an introductory calculus course in the Fall semester of 2021, which was the third course in a sequence. I was shocked at how many of my students struggled with topics in algebra, taught online over the previous two semesters. So I adjusted my teaching, and spent much more time talking through the algebraic steps of the problems in class. By the end of the semester, many of those students had mastered algebra as well as the intended course content. I asked the course coordinator whether our traditional grading scheme was appropriate, when so many students had shown so much improvement in their first offline math class in college. We settled on a more equitable grading scheme that emphasized what students had accomplished by the end of the semester, rather than poor scores on early tests.

I try to grow as a teacher in some way every year. Last year, when I designed a 100 level elective course *The Mathematics of Voting*, I learned about the rich diversity of teaching tools employed often in other fields, but rarely in math. I taught that class using reflections, roleplay, and small-group-in-large-group discussions. I think the most valuable thing I learned was not merely how to use these tools in a classroom, but that they have different goals than lectures or most problem sets. Reflections give students the opportunity to organize their thoughts or evaluate their work. These are processes I am emphasizing now that I am teaching proof writing, particularly by assigning short quizzes with second drafts. Roleplay is experiential, and if there's anything I learned teaching *The Mathematics of Voting*, it's the value of experiencing something that might be hard to understand on paper. I think, in the coming year, that's the aspect of teaching I most want to explore.

I became a mathematician because I think math is worthwhile, but also because it is hard. I like to be challenged, and I enjoy the catharsis of finally solving a tough problem. My absolute favorite thing as a teacher is when a student discovers that math is meaningful, and fascinating, and something they can excel at even if it's hard. I also acknowledge that that's not always what a student wants or needs. Whatever the case, I strive to understand and adapt to a student's or a classrooms' needs, and focus on their successes rather than my own expectations.

3 Documentation of Teaching Experience

In this section, I describe courses for which I have been the primary instructor and administrator. These are listed reverse chronologically. The table below outlines the courses I have taught, listed by how recently I taught the course, the total number of semesters I taught that course, the purpose of the course, the format, and whether or not I played a significant role in designing or developing the course.

Taught	Course Name	Times	Purpose	Format	Developer
2022	Discrete Math	1	Introduction to Proofs	Mid-sized lecture	✓
2022	The Mathematics of Voting	1	Elective	Small Classroom	✓
2021	Introduction to Calculus with Applications	1	Support	Small Classroom	
2021	Introduction to Finite Math Part 2	1	General Education	Zoom	
2020	The Mathematics of Decision and Beauty	5	General Education	Small Classroom Zoom	✓
2019	Finite Mathematics	2	General Education	Large Lecture	
2017	Introduction to Finite Math Part 1	1	General Education	Large Classroom	
2016	Basic Algebra for Finite Mathematics	1	Developmental	Small Classroom	

3.1 Discrete Math at Williams College

I am currently teaching a course in Discrete Mathematics that serves as an introduction to proof writing for math and computer science majors.

3.1.1 Documentation

A syllabus and midterm for this class, both written by me, appear in the appendices.

3.1.2 Topics and Resources

The topics of the course are

- Set theory
- Mathematical notation
- Logical statements and arguments
- Elementary number theory
- Direct proofs
- Proofs by counterexample
- Sequences
- Recursion
- Proofs by induction
- Functions
- Relations
- Counting
- Probability
- Graphs.

We use the textbook *Discrete Mathematics with Applications* by Susanna Epp.

3.1.3 Structure

Due to the size of the sections, classroom layout, and pace of the class I am primarily lecturing in this course. I do give students time in class to work through problems in order to break up the lectures, and help them to stay engaged.

The course has weekly homework, which is typically a problem set. Every week there is also a quiz; this is a 15 minute proof writing exercises that the students can complete online at their convenience. Class TAs give the students feedback on their quiz submission, and students are asked to resubmit if their proof had serious flaws. There are also two take-home exams, which are structured similarly to homework but are cumulative. There is also an in-class midterm, and an in-class final. The final grade is calculated as a weighted average of these:

Homework	25%
Quizzes	15%
Test 1, take-home	10%
Midterm, in-class	15%
Test 3, take-home	10%
Final, in-class	25%

Previous iterations of this course had made in-class tests account for the majority of a students' final grade (previously 50%, 75%, or 80%). I felt it was important to emphasize work students could complete at their own pace, so I implemented this grading scheme after conferring with previous instructors.

3.1.4 Changes for the Future

I will most likely teach this course again in the Spring. In addition to small changes to individual classes or problem sets, I have decided to restructure the schedule of the class to spend more time on introductory material than I scheduled this semester. I will also generally be looking for more problems I can ask students to work through during class.

3.1.5 Student Feedback

From a post-midterm survey. I beyond appreciate how organized your lecture notes and preparation is. With little-to-no downtime in the lecture itself, the learning feels efficient in a way that makes the morning class go by quickly and helps me remain interested and engaged.

From a post-midterm survey. I like how understanding the professor is.

From a post-midterm survey. I like the new spacing between problem sets, it gives plenty of time to get them done. I also like that we have some take home exams and some in class, the variety is nice and makes it so there is not too much pressure on any one exam. [3]

[3] As this student's comment indicates, I changed the scheduling of problem sets a few weeks into the course to better fit the students' schedules.

3.2 The Mathematics of Voting at Collins LLC, Indiana University

I taught this course in the Spring semester of 2022. This was an elective course supported by the Collins Living Learning Center, an organization within Indiana University with a liberal arts focus. This was taught as a math course, but was taken by students from a variety of majors.

3.2.1 Documentation

A syllabus and assignment for this class, both written by me, appear in the appendices.

3.2.2 Topics and Resources

Broadly this class covered a variety of voting methods, redisticting methods, and fairness principles for these processes. We also discussed complexity, gerrymandering, strategic voting, chaotic systems, and game theory. How to communicate about math was a recurring theme. A full list of topics are included in the syllabus in the appendices.

3.2.3 Structure

I taught this course in a small classroom, and as much classroom time as possible was used on activities or problem sets. One midterm was scheduled toward the end of the semester, with the intent that the last several weeks of class would be spent on projects.

Due to a worker strike on campus toward the end of the semester, students' final grades were an average of their scores on completed assignments.

3.2.4 Changes for the future

I essentially designed this course from scratch, as part of the CIRTL Network's TYRIT (Transforming Your Research Into Teaching) program. This program did not assume that a textbook would exist for the course, and so I picked a book after deciding what topics to cover: *The Mathematics of Politics* by E. Arthur Robinson and Daniel H. Ullman. I felt the book did not align closely enough with my goals for the course, and I published quite a lot of class notes to account for the discrepancies.

Were I to teach this course again, I would be inclined to write more complete lecture notes and forego the textbook, or to adjust the course to align with the book *Mathematics and Politics: Strategy, Voting, Power, and Proof* by Allison Pacelli and Alan D. Taylor.

3.2.5 Feedback from Students

Due to support from students in this class, I received the Carl Ziegler Teaching Award, "for outstanding teaching in the Collins community."

Taken from the Collins LLC Annual Report. Collinsites appreciated his Mathematics of Voting class for the ways that it used interactive teaching to make math accessible. They also lauded Daniel's kindness, availability, and generosity.

3.3 Introduction to Calculus with Applications, at Indiana University

I taught this course in a small classroom in the fall semester of 2021. This course is meant to support students going into STEM. It is the third course in a sequence for students in the Groups program, a program targeted at first-generation and low-income students, and meets students' general education math requirement.

3.3.1 Topics and Resources

The topics in this course included

- polynomial, rational, exponential, and logarithmic functions
- limits
- continuity
- derivatives and techniques for differentiation
- finding extrema to solve optimization problems
- related rages
- antiderivatives
- the fundamental theorem of calculus.

The textbook used was *Calculus with Applications* by Lial.

3.3.2 Remark

This course was taught with a parallel section taught by another instructor, and a faculty coordinator for the two sections.

I observed in this course that many students who struggled terribly with basic algebra at the beginning of the semester still managed to master most of the course content by the end of the semester. I hypothesized that disruptions due to the Covid 19 pandemic had rendered their previous two years of mathematical instruction far less effective, and that these students were taking their first rigorous math class in years. I believe that many of these students struggled with algebra not because they had never learned it, but because they were out of practice.

In addition to adjusting my teaching to accommodate students with weaker algebra skills, I requested of the course coordinator that we adopt a grading scheme that was less punishing of students who showed dramatic improvement over the semester. He agreed, and I believe the grading scheme we adopted was fairer given the context.

I have since discussed this phenomenon with other calculus instructors from other institutions, who have generally had similar experiences and felt similarly.

3.4 Introduction to Finite Mathematics Part 2, at Indiana University

I taught this course on Zoom in the spring semester of 2021. This is a general education course that goes at a slower pace to the standard course Finite Mathematics, and covers the second half of the content of that course.

3.4.1 Topics and Resources

The topics in this course included

- matrix arithmetic
- matrix row reduction to solve systems of linear equations
- linear programming
- Markov chains.

This course is based on the textbook *Finite Mathematics* by Daniel P. Maki and Maynard Thompson.

3.4.2 Remark

This course had been redesigned a few years prior to the pandemic to be driven almost entirely by in-class group work around a series of problem sets. While I think that approach was appropriate for the purposes of this class, it was much less effective online.

Along with instructors of parallel sections, we reorganized the course to deemphasize group work, which seemed to work better for students.

3.4.3 Feedback from Students

Comment on a course evaluation. Daniel was always willing to answer any and all questions, as many times as it took for you to understand. He was very understanding with us in terms of timelines for stuff and would push back due dates if needed. He also asked for our input on a number of issues which was nice. Additionally, his office hours were super helpful. You could go and tell him what you needed help with and he would go through examples until you had a better understanding. He always encouraged us to attend office hours, PASS, and other tutoring services. He also advocated for the use of CAPS when recognizing a number of us may have been feeling down/stressed. [4]

[4] CAPS is the university counselling center. I try to advertise such university resources in my classes, and this semester it seemed particularly prudent.

3.5 The Mathematics of Decision and Beauty, at Indiana University

I taught this course several times between 2017 and 2020, in a small classroom except for the last time which was online. This course surveys topics in mathematics and satisfies most majors' general education math requirement, and is mostly taken by students who have weak algebra backgrounds, and humanities students with a specific interest in one of the course topics.

3.5.1 Documentation

A syllabus, scheduling guide for instructors, exam, sample class notes, and samples of student work, all created by me, appear in the appendices.

3.5.2 Topics and Resources

The topics in this course included

- music theory
- voting theory
- graph theory
- game theory
- 2-dimensional projection of 3-dimensional objects
- groups of symmetries of figures and solids.

Most of the materials for this course, including class notes, problem sets, and a problem bank, were created by me and were used for several years. When Indiana University hired a new coordinator for the course, he created new notes, but continued to use the problem sets I created.

3.5.3 Feedback from Students

Comment on a course evaluation. As someone who has taken a lot of my classes over the years (only 1 at IU), I can honestly say that Daniel is the best instructor I've ever had. He's very personable and reasonable and explains everything in a way that's easy to understand.

Comment on a course evaluation. Daniel was very approachable and very patient when I was lost on a topic.

Comment on a course evaluation. Patient. Always checked with students for comprehension, offered office hours daily, sense of humor.

Comment on a course evaluation. I felt like our professor was very helpful in making sure the topic was understood. The has been by far the best math class I have had in my educational career. Thank you!

Comment on a course evaluation. He's the most relatable, understanding, caring, and "real" math professor that I've had at IU. A lot of complaints are always made towards the instructors in more math related courses that regard their ability to connect to students. Daniel is one of the few that had no issue with this, and made each meeting for class a helpful and informative time.

Comment on a course evaluation. He was really interested in the topics and really accommodating. Whenever the class at large had issues with homework or topics he'd give us extra time and go back over things to make sure we understood what we were doing. He changed his office hours to better suit the class and he was really available to help us.

Comment on a course evaluation. Daniel was one of the best instructors I've had at IU. He was well organized, set clear standards and expectations, and was reasonable in everything he did. As to the course, it was so much more enjoyable than any math class I've ever taken. I'm "not a math person", but I actually liked being in the class, which surprised me. I found the concepts interesting. I wish that there were more classes like this one.

Comment on a course evaluation. I used to hate and DREAD math (I failed Finite 3 times) but this class was my favorite course in my 5 years of college. It was challenging but also interesting and even fun. Daniel is super knowledgeable and kind. He goes above and beyond for his students...

Unsolicited Comment. Thank you for being the best math instructor I've ever had. I've always loved math, but I usually end up having to teach the lessons to myself, but I was able to learn just from class/your notes. Thank you.

Unsolicited Comment. Daniel, Thank you for an awesome semester. The OCQs are anonymous, but you are honestly one of the most hardworking, caring & supportive instructors I have ever had. Thank you!!

3.6 Finite Mathematics, at Indiana University

This is the flagship general education math course at Indiana University. I taught the class once in a small classroom in 2016, and once in a large lecture in 2019.

3.6.1 Documentation

A test that I wrote for this class appears in the appendices.

3.6.2 Topics and Resources

The topics in this course included

- set theory
- counting
- probability
- matrix arithmetic
- matrix row reduction to solve systems of linear equations
- linear programming
- Markov chains.

This course is based on the textbook *Finite Mathematics* by Daniel P. Maki and Maynard Thompson.

3.6.3 Feedback from Students

Comment on a course evaluation. Mr. Daniel Condon was an excellent instructor. He was approachable when we had questions and was knowledgeable and passionate about the subject matter. This style of teaching was excellent for those who are not mathematically inclined, and he never ran out of patience or manners.

Comment on a course evaluation. Daniel makes the lecture portion of the class not boring. It's hard to focus and be engaged at 8 am but I find that with Daniel as the teacher I don't hate getting up early for class...

Comment on a course evaluation. I don't really have a favorite part of the course, but Prof Condon was always funny and kept the room light. It made it a little easier to go to 8 am Finite.

Comment on a course evaluation. Mr. Condon gave in depth explanations of each topic, giving us the what and why of each and every step, in such a helpful manner that I myself was able to help many of my friends who's instructors were not so helpful. I feel very lucky to have had Mr. Condon as my instructor.

Comment on a course evaluation. Professor Condon always brought energy and enthusiasm to class which made a dreary subject light and comparatively better than what I imagine the class could have been. Also, he always arrives early to class to answer any student questions in addition to his typical office hours!

Comment on a course evaluation. Daniel Condon was a great instructor who thoroughly went through each step of all of the content this semester. Even though some of his steps between how to solve problems were obvious sometimes, I appreciate that he took the time to do them anyway for the sake of clarity.

3.7 Introduction to Finite Mathematics Part 1, at Indiana University

I taught this class in a large classroom in 2018. This is a general education course that goes at a slower pace to the standard course Finite Mathematics, and covers the first half of the content of that course.

3.7.1 Documentation

An in-class quiz I gave in this class appears in the appendices.

3.7.2 Topics and Resources

The topics in this course included

- set theory
- counting
- probability

This course is based on the textbook *Finite Mathematics* by Daniel P. Maki and Maynard Thompson.

3.7.3 Feedback from Students

Comment on a course evaluation. This instructor was patient with the class and always asked the students if the taught material was understood by everyone. He always allowed students to comment and ask questions of clarification or better explanation.

3.8 Basic Algebra for Finite Mathematics, at Indiana University

I taught two sections of this half-semester class in 2016. The course is developmental, and intended for students who will take the two part Finite Math sequence, but who are not prepared to start it.

3.8.1 Topics and Resources

The topics in this course included

- set theory
- linear algebra
- counting
- probability.

The textbook used is *Basic Algebra for Finite Math* by Neece.

3.8.2 Feedback from Students

Comment on a course evaluation. His personality and connection he formed with his students.

Comment on a course evaluation. The pace of this course and Daniel made sure the class had some fun and really understood the material well.

Comment on a course evaluation. Daniel explains things very clearly and simply. This was good when learning something new, but frustrating when I already had a good understanding of the topic.

Comment on a course evaluation. The pace was perfect and the instructor was fantastic at explaining course material and keeping students engaged.

4 Mentorship and Professional Development

4.1 The Directed Reading Program

In the Directed Reading Program, advanced undergraduates studying math are paired with graduate student mentors to participate in a reading course for one semester. Students read a math book in a topic not regularly taught at Indiana University. They meet with their mentors for at least one hour a week to discuss the topics of that book, and they give a presentation on a topic from their book at the end of the semester.

I was a mentor for this program in the Fall 2018 semester and Spring 2019 semester with two different students who both read Volume 1 of *Winning Ways for your Mathematical Plays* by Berlekamp, Conway, and Guy, a seminal but fairly accessible text on combinatorial game theory.

In the Spring 2021 semester I worked with a student who had broad interests in discrete math. We spent a while studying Conway's Game of Life, discussed some other topics in combinatorics briefly, and then spent the rest of the semester reading *The Game of Cops and Robbers on Graphs* by Bonato and Nowakowski.

Our discussions were mostly about the texts we were reading, but not restricted to it. For example, in all three semesters we discussed games not from the text. We also discussed related topics regarding math and being a math major, such as non-combinatorial game theory and how to write a proof. We also workshopped their math talks for the end of the semester.

4.2 The Peer Mentoring Program

In 2020 I helped found a peer mentoring program for graduate students at Indiana University. Our primary goal was to help incoming students make connections within the department, with the purpose of making the department more inclusive.

I helped organize the program in its first year, and in the 2021-2022 academic year I am serving as a peer mentor for one incoming student.

4.3 Teacher Training Courses

The Indiana University math department offers two teacher training courses. The first (M595) is about teaching undergraduate college math, which I took in my first year at IU. The second (M596) focuses specifically on teaching *M106 The Mathematics of Teaching and Beauty*; I have assisted in teaching this course by giving a talk about teaching voting theory.

4.4 2020 Summer Teaching Workshop

In the summer semester of 2020 I reached out to IU's Center for Innovative Teaching and Learning to request help organizing a teaching workshop on inclusive teaching methods. This one hour workshop was ultimately led by a member of CITL, was available to all math graduate students, and discussed inclusion, justice, and equity in teaching.

4.5 TYRIT

The Transforming-Your-Research-Into-Teaching workshop is a national seminar through the CIRTL network that runs for 6 weeks. I participated in the 2021 seminar in order to better develop the Mathematics of Voting course I was designing. I was surprised by the diversity in teaching methods employed by other fields, and I have done my best to incorporate these into my voting theory course, as appropriate.

4.6 First3 Course Design Workshop

The First3 program is aimed at new faculty at Williams College, and offers a 6 week course design workshop taught by Betsy Burris. I found this workshop extremely helpful, as it offered a very different perspective than the TYRIT workshop. Using the methods I learned, I was able to prepare my teaching for the fall extremely efficiently. I believe that as a teacher, time management not only benefits me, but makes my courses better, and I plan to use the methods taught in this workshop in the future.

4.7 Awards

I was awarded the **David A. Rothrock Teaching Award** in the spring semester of 2018, and the **David A. Rothrock Teaching Fellowship** in the spring semesters of 2020 and 2021, for "excellence [in] teaching of mathematics," both at Indiana University.

In 2022, I was awarded the **Carl Ziegler Teaching Award** for excellent teaching at the Collins Living Learning Center at Indiana University, for the course I designed *The Mathematics of Voting.*

Appendices: Materials and Documents

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Finite Math Test

The following 5 pages include the problems from a test written by me for my section taking M118 Finite Mathematics. Some of the stylistic choices in this test are simply in keeping with standard test design for this class in the department: a multiple choice test; wrong answers are based on expected mistakes; the answer choice "none of the above" is never intended to be the correct answer, which is explained to students; the difficulty is typical of the third exam in this class; some of the terminology, if a little informal, is typical of the course. If I rewrote this test, I would not deviate from the department style.

I do think there were some missed opportunities for teaching. For example, problem 1 is a linear programming problem typical of the course, whereas problem 9 is an atypical linear programming problem. Juxtaposing these problems could clarify the writer's intent so that students who were lost in the original version of the test might instead have learned something new with the proposed change.

Problems 12 and 18 have a similar relationship of escalating abstraction and difficulty, and might also benefit from juxtaposition, though I think the effect with these problems would be less dramatic.

1. Chuck's Dog Chow comes in two varieties: Good Doggo and Healthy Pupper. Each ounce of Good Doggo chow contains 2 gram of proteins and 12 grams of filler. Each ounce of Healthy Pupper chow contains 5 grams of protein and 10 grams of filler. How many ounces of each type of chow can be made in order to use up 500 grams of Protein and 1400 grams of Filler?

   (a) no Good Doggo, and 100 oz of Healthy Pupper
   (b) 100 oz of Good Doggo, and 60 oz of Healthy Pupper
   (c) 60 oz of Good Doggo, and 50 oz of Healthy Pupper
   (d) 50 oz of Good Doggo, and 80 oz of Healthy Pupper
   (e) 150 oz of Good Doggo, and no Healthy Pupper
   (f) None of the Above

2. Solve for A:

$$A + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$$

   (a) $A = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$
   (b) $A = \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}$
   (c) $A = \begin{pmatrix} 0 & -1 \\ 2 & 0 \end{pmatrix}$
   (d) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
   (e) $A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$
   (f) None of the Above

3. The following is an augmented matrix for a system of equations with variables w, x, y, z.

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 2 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right)$$

   Which of the options below are the solution set for this system?

   (a) $w = 2,\quad x = 3,\quad y = y,\quad z = -3$
   (b) $w = 1 - y,\quad x = 1 - \frac{y}{3},\quad y = y,\quad z = -3$
   (c) $w = 2 - 2y,\quad x = 3 - y,\quad y = y,\quad z = -3$
   (d) $w = 2,\quad x = 3,\quad y = 0,\quad z = -3$
   (e) $w = 2,\quad x = 3,\quad y = \{2, 3\},\quad z = -3$
   (f) None of the Above

4. Solve for X in the matrix equation

$$AX + BX = C$$

   where $A = \begin{pmatrix} 4 & 6 \\ -3 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -6 \\ 3 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 5 & 0 \\ 10 & -5 \end{pmatrix}$.

   (a) $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$
   (b) $A = \begin{pmatrix} 10 & 5 \\ 15 & 0 \end{pmatrix}$
   (c) $A = \begin{pmatrix} 0 & -5 \\ 5 & -10 \end{pmatrix}$
   (d) $A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$
   (e) $A = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$
   (f) None of the Above

5. Find a value of k so that the matrix below has no inverse:

$$\begin{pmatrix} 1 & 2 & k \\ 0 & 1 & 1 \\ 1 & 1 & 3 \end{pmatrix}$$

(a) $k = 0$
(b) $k = 1$
(c) $k = 4$
(d) There are multiple values of k for which this matrix is not invertible.
(e) This matrix is invertible for any value of k
(f) None of the Above

6. Find the inverse of the matrix $A = \begin{pmatrix} 2 & 1 \\ 8 & 3 \end{pmatrix}$.

(a) $A^{-1} = \begin{pmatrix} 3 & 1 \\ 8 & 2 \end{pmatrix}$
(b) $A^{-1} = \begin{pmatrix} -1.5 & .5 \\ 4 & -1 \end{pmatrix}$
(c) $A^{-1} = \begin{pmatrix} -3 & 1 \\ 8 & -2 \end{pmatrix}$
(d) $A^{-1} = \begin{pmatrix} 1.5 & -.5 \\ -4 & 1 \end{pmatrix}$
(e) $A^{-1} = \begin{pmatrix} 3 & -1 \\ -8 & 2 \end{pmatrix}$
(f) None of the Above

7. A small economy has the following technology matrix, where the first row corresponds to petroleum, and the second row corresponds to cheese.

$$A = \begin{pmatrix} .8 & .9 \\ .1 & .5 \end{pmatrix}$$

How many units of petroleum are necessary to fill an external demand for 5 units of cheese?

(a) 0

(b) 50

(c) 100

(d) 250

(e) 450

(f) None of the Above

8. What is the y-intercept of the line that is parallel to the line $y = \frac{3}{2}x + 7$ and contains the point $(8, 2)$?

(a) -14
(b) -10
(c) -5
(d) -2
(e) 7
(f) None of the Above

9. Melf's Muffins makes peanut-crunch muffins and double-chocolate muffins. A peanut-crunch muffin contains an ounce of peanuts and a tablespoon of chocolate chips. A double-chocolate muffin contains no peanuts, but does contain three tablespoons of chocolate chips. Melf plans to cook 100 muffins, using 1 gallon (256 tablespoons) of chocolate chips. How many ounces of peanuts will he need?

   (a) 22
   (b) 64
   (c) 78
   (d) 100
   (e) 156
   (f) None of the Above

10. What are the dimensions of the matrix resulting from the calculation below:

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 1 \\ 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & 2 & 2 \\ -1 & 3 & 1 & 3 \\ -1 & 2 & 1 & -4 \end{pmatrix} - \begin{pmatrix} 1 & 2 & 2 & 2 \\ -1 & 3 & 1 & 3 \\ -1 & 2 & 1 & -4 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 1 \\ 2 & 0 & 0 \end{pmatrix}$$

   (a) 3×4
   (b) 3×3
   (c) 4×4
   (d) 4×3
   (e) This calculation is not well defined
   (f) None of the Above

11. Solve the following matrix equation:

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 4 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}$$

   (a) $x = 1, y = 1, z = 3$
   (b) $x = 0, y = 2, z = 0$
   (c) $x = 1, y = 2, z = 2$
   (d) There are infinitely many solutions
   (e) There is no solution
   (f) None of the Above

12. The value of the 1998 Ford Torus has depended linearly on the year since 2010, when it was valued at \$5000. If the car was valued at \$3500 in 2014, how much was the care worth in 2016?

   (a) \$2000
   (b) \$2250
   (c) \$2500
   (d) \$2750
   (e) \$3000
   (f) None of the Above

13. Solve the following system of equations:

$$\begin{array}{rcrcrl} x & + & y & + & z & = 4 \\ 2x & - & y & - & 2z & = -5 \\ x & + & y & & & = 0 \end{array}$$

(a) $x = 1, y = 1, z = 2$
(b) $x = 1, y = -1, z = 4$
(c) $x = 2, y = -2, z = 4$
(d) The system has infinitely many solutions.
(e) The system has no solution.
(f) None of the Above

14. Row reduce the following matrix to the correct reduced form:

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 1 & 2 & 3 & 7 \\ 2 & 6 & 2 & 0 \end{array}\right)$$

(a) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$
(b) $\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$
(c) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & -1 \end{array}\right)$
(d) $\left(\begin{array}{ccc|c} 1 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$
(e) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right)$
(f) None of the Above

15. What line is parallel to the line

$$3x + 4y = 12$$

and contains the point $(7, 3)$?

(a) $3x + 4y = 3$
(b) $3x + 4y = 7$
(c) $3x + 4y = 12$
(d) $3x + 4y = 33$
(e) $3x + 4y = 45$
(f) None of the Above

16. Find the entry of the second row and fourth column of the following matrix product:

$$\begin{pmatrix} 10 & 0 & 1 \\ 2 & -2 & 3 \\ 4 & -7 & 13 \end{pmatrix} \begin{pmatrix} -1 & 1 & 2 & 4 \\ 1 & -2 & 4 & -8 \\ \frac{1}{3} & \frac{1}{9} & -\frac{1}{9} & -\frac{1}{3} \end{pmatrix}$$

(a) -7
(b) -3
(c) $10\frac{1}{9}$
(d) 23
(e) $39\frac{2}{3}$
(f) None of the Above

17. A village produces hemp rope and fish. The fishers of the village consume on average 1 foot of rope for every 100 fish they catch, and eat 2 fish in the process. The ropemakers eat on average 1 fish for every 10 feet of rope they make. Which of the matrices below could reasonably be a technology matrix modeling the local economy? (the units used are 10 feet of rope and 100 fish)

(a) $A = \begin{pmatrix} 0 & .1 \\ .01 & .02 \end{pmatrix}$
(b) $A = \begin{pmatrix} 0 & .01 \\ .1 & .02 \end{pmatrix}$
(c) $A = \begin{pmatrix} 1 & -.1 \\ -.01 & .98 \end{pmatrix}$
(d) $A = \begin{pmatrix} 1 & -.01 \\ -.1 & .98 \end{pmatrix}$
(e) $A = \frac{1}{.979} \begin{pmatrix} .98 & .1 \\ .01 & 1 \end{pmatrix}$
(f) None of the Above

18. The 1990 Dodge Ram cost \$16,000 when it was first released, and depreciated in value linearly. The 1990 Ram cost \$11,000 in 1995. The 1990 Chevy Malibu initially cost twice as much as the Ram, but depreciated in value three times as quickly. In what year did the vehicles have the same value?

(a) 1990
(b) 1995
(c) 1998
(d) 2001
(e) 2005
(f) None of the Above

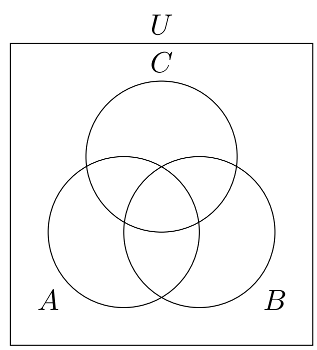
Finite Math Part 1 Quizzes

These quizzes were turned in at the beginning of class, after which I showed students how to work through the problems using a projector. Each quiz had three questions ordered by increasing difficulty, covering content from the previous lecture, with the intent that almost every student would be able to complete the first two problems accurately and quickly, and that the last problem would be more challenging.

Quiz. Section 1.3
Problem 1. If $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6, 8, 10\}$ then what are $n(A), n(B), n(A \cap B)$, and $n(A \cup B)$?

Problem 2. In a universal set U containing the sets A and B, suppose $n(U) = 100, n(A \cap B) = 30, n(A \cap B') = 30$, and $n(B') = 50$. What is $n(A \cup B)$?

Problem 3. Let U be a universe set containing the sets A, B, C. Let $n(A \cap B \cap C) = 0, n(A \cap B) = 5, n(A \cap C) = 5, n(B \cap C) = 6, n(A) = 12, n(C) = 13, n(B) = 14$. What is $n(A \cup B \cup C)$?



Quiz. Section 3.1
Problem 1. The forecast predicts a 40 % chance of rain. What is the probability it does not rain? Give your answer as a percent.

Problem 2. On a particular exam, $\frac{1}{3}$ of students failed and $\frac{1}{10}$ of students received an A. What is the probability that a randomly selected student passed the test, but did not receive an A?

Problem 3. The forecast predicts rain with probability .6, high wind with probability .7. More specifically, there is a .5 chance that there will be both rain and high wind. What is the probability that it will just be rainy, or just windy, but not both rainy and windy.

My intent at the time was that the challenging problem would motivate students to keep up with the coursework, and would also provide them with more meaningful feedback than the easier problems. However, if I found myself using quizzes to motivate attendance now, I would make the quizzes open note and structure them as review of the previous topic, or preparation for an upcoming topic. For example:

Quiz. Section 1.3 (Revised)
Recall that $n(S)$ denotes the number of elements in S, that $A \cap B$ is the set of all elements which are in both A and B, and $A \cup B$ is the set of all elements which are in either A or B or both.

Problem 1. If $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6, 8, 10\}$ then what are $n(A), n(B), n(A \cap B)$, and $n(A \cup B)$?

Problem 2. In a universal set U containing the sets A and B, suppose $n(U) = 100$, $n(A \cap B) = 30$, $n(A \cap B') = 20$, and $n(B') = 60$. Draw a Venn diagram for the sets A and B in U, and place the numbers 30 and 20 in the appropriate places in the diagram.

Problem 3. Copy your Venn diagram from problem 2, and fill in the rest of the diagram.

Quiz. Section 3.1 (Revised)
Recall that the probabilities of all disjoint outcomes in a scenario should add up to 1, or 100%.

Problem 1. Identify which of the following situations is impossible, and explain why.

- The forecast predicts a 40% chance it will rain, and a 60% it will not rain.
- In one math class, the probability of a student receiving an A is .2, the probability of receiving a B is .3, the probability of receiving a C is .4, the probability of receiving a D is .1, and the probability of receiving an F is .2.

Problem 2. On a particular exam, $\frac{1}{3}$ of students failed and $\frac{1}{10}$ of students received an A. What is the probability that a randomly selected student passed the test, but did not receive an A?

Problem 3. The forecast predicts rain with probability .6, high wind with probability .7. More specifically, there is a .5 chance that there will be both rain and high wind. What is the probability that it will be just rainy, or just windy, but not both rainy and windy?

hint: Draw a Venn diagram, and label each region in the diagram with a probability. All four probabilities should add up to 1.

Decision and Beauty Scheduling Guide

The following four pages contain a sample of a guide I created for instructors of M106. This document was meant to help parallel instructors teach uniformly.

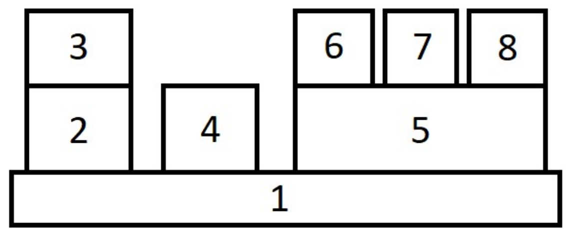
Coordinating the Graph Theory module

The notes have these sections:

1. Graphs and Digraphs
    a. Graphs, vertices, edges, degree
    b. Weighted graphs
    c. Directed graphs
2. Euler Circuits
    a. Walks
    b. Cycles
    c. The Konigsberg Bridge Puzzle
    d. Eulerian Circuits and Paths
3. Eulerian Graphs and Eulerization
    a. Eulerian Graphs
    b. Connectedness
    c. Eulerization by duplicating edges
    d. Eulerization by removing edges
4. Graph Drawings and Isomorphisms
    a. Graphs vs. Graph Drawings
    b. Isomorphic Graphs
    c. Planarity
5. Special Types of Graphs
    a. Complete Graphs
    b. Cycle Graphs
    c. Subgraphs
    d. Trees
    e. Bipartite Graphs
6. Hamiltonian Circuits
    a. Hamiltonian Circuits
    b. Graphs without H. Circuits
    c. Traveling Salesman Problem
    d. Nearest Neighbor Algorithm
    e. Sorted Edges Algorithm
7. Spanning Trees
    a. Trees
    b. Spanning Trees
    c. Minimum spanning trees
    d. Kruskal's Algorithm
8. The Chromatic Number
    a. Proper Coloring
    b. N-colorable
    c. Chromatic Number
    d. Clique and Clique Number
    e. Greedy Coloring
    f. Scheduling Problems

Each of these sections are intended to be accessible to students. They each have an associated homework assignment consisting primarily of routine problems. Each section in the notes also ends in some exercises, most of which are more thought provoking than the standard assignments – documents exist to turn some of these problems into longer in-class exercises. The red sections are what you are expected to cover in a typical course. The information in the other sections would typically filter into your course *anyway*, so you can either treat them as independent topics (as they are written in the notes), or just cover those ideas as they become relevant.

The dependency of the sections is approximately:



In addition to the lecture notes, the following resources are available to you:

- An assignment for each section
- Additional assignments for certain sections
- Two exam practice assignments
- An exam problem bank
- Old exams

A specific account of what documents exist is given on the next page.

However you decide to organize your course, consistency is important. You and the other instructors should agree on exactly which topics to include on the exam, before you start teaching the section. One good way to do this would be to draft the exam before this section starts. Or, you might agree on a set of assignments to assign and build the exam around.

Available Documents

You have access to a number of assignments and other resources, although you need not use all of them. You should probably assign the worksheets which are marked in red, as well as an additional couple of worksheets, and an exam practice problem set. For most of these documents, both the .pdf and .tex are available.

Notes	
main.pdf	These are the graph theory notes for the course
Konigsberg2.png, NewEngland.png	These are used in the notes
Assignments	
1. Graphs and Digraphs.pdf	Section 1 worksheet
2. Euler Circuits.pdf	Section 2 worksheet
2. Euler Circuits – Konigsberg.pdf	Section 2 additional worksheet
3. Euler Graphs and Eulerization.pdf	Section 3 worksheet
4. Graph Drawings and Isomorphisms.pdf	Section 4 worksheet
5. Special Types of Graphs.pdf	Section 5 worksheet
6. Hamiltonian Circuits.pdf	Section 6 worksheet
7. Spanning Trees.pdf	Section 7 worksheet
8. Graph Coloring.pdf	Section 8 worksheet
8. Graph Coloring – Scheduling Problems.pdf	Section 8 additional worksheet
Exam Resources	
Exam Prep.pdf	A set of practice questions for the exam
Exam Practice Questions.pdf	Additional practice questions
Exam Sample Questions.pdf	A problem bank of exam questions: you are encouraged to write your exam using these.
Old Exams	
Graph Exam Dec 2017 Condon.pdf	Old graph theory exam
Graph Exam Dec 2017 Hussung.pdf	Old graph theory exam
Meta	
Instructor Readme.pdf	.docx file also available
Graph Theory Section Dependence.png	

All pdfs except for the old exams come with .tex files.

The following gives a sample schedule for during the semester (50 minute lectures):

Day	Sections	Learning Objectives	Assignments
1	Graphs and Digraphs	Vocab: graph, vertex, edge, digraph, degree Use graphs to represent real world situations	Section 1 Worksheet
2	Konigsberg	Vocab: walk, circuit Discover a characterization of Eulerian graphs	Konigsberg Worksheet
3	Eulerian Graphs	Vocab: Eulerian, connected, component Identify which graphs have Eulerian Circuits or Paths	Section 2 Worksheet
4	Eulerization	Vocab: Eulerization Eulerize graphs by duplicating edges	
5	Eulerization	Eulerize graphs by removing edges Identify methods of Eulerization appropriate to a particular problem	Section 3 Worksheet
6	Graph Drawings	Vocab: drawing, equal, isomorphism, planar Distinguish between graphs and drawings Identify isomorphic graphs Be able to redraw graphs differently	Section 4 Worksheet
7	Special Graphs	Vocab: tree, leaf, cycle, complete graph, bipartite, subgraph Identify certain special graphs and some of their properties	Section 5 Worksheet
8	Hamiltonian Circuits	Vocab: Hamiltonian Circuit, cut vertex, the Traveling Salesman Problem, algorithm, greedy Identify when a graph clearly has no Hamiltonian Circuit Apply the Nearest Neighbor Algorithm	
9	Hamiltonian Circuits	Apply the Sorted Edges Algorithm Identify the failings of the NN Algorithm and SE Algorithm	Section 6 Worksheet
10	Spanning Trees	Vocab: Spanning tree, minimum spanning tree Know when to use a minimum spanning tree Apply Kruskal's Algorithm	Section 7 Worksheet
11	Graph Coloring	Vocab: proper coloring, chromatic number, clique, clique number Identify a clique on a graph Find the chromatic number of a graph Apply the greedy coloring algorithm	Section 8 Worksheet
12	Scheduling Problems	Practice coloring and solve scheduling problems	Scheduling Problems Worksheet
13	Review		
14	Review		
15	Exam		

The following gives a sample schedule for during the summer (120 minute lectures):

Day	Sections	Learning Objectives	Assignments
1	Graphs and Digraphs Konigsberg	See above	Section 1 worksheet Konigsberg Worksheet
2	Euler Circuits Eulerization		Section 2 Worksheet
3	Eulerization		Section 3 Worksheet
4	Graph Drawings Special Graphs		Section 4 Worksheet Section 5 Worksheet
5	Hamiltonian Circuits		Section 6 Worksheet
6	Spanning Trees		Section 7 Worksheet
7	Graph Coloring and Scheduling Problems		Section 8 Worksheet Scheduling Problems Worksheet
8	Review		
9	Review and Exam		Exam

Decision and Beauty Sample Class Notes

The following six pages contain notes I wrote for the graph theory unit of *M106 The Mathematics of Decision and Beauty*. These are from the instructor edition, which differs from the student edition by the inclusion of comments in the margins.

With the benefits of having taught the course with these materials, and having seen alterations made by later instructors, I have the following reflections:

- I stand by my choice to give informal definitions rather than set-theoretic ones, something that some later instructors changed.
- Problem 2 is very important because it catches a common mistake that students make - thinking that the crossing of edges in the middle of the graph constitutes a vertex.
- Problems 3-5 are both conceptually challenging and technically challenging[5] for many students. I would only give these problems as classwork, not homework, and the instructor edition could benefit from a note to this effect. The problem might be better if one of the people was removed.
- Problem 7 requires some tricky arithmetic. While one might feel college students should have no trouble with arithmetic, arithmetic isn't the point of this class, and the instruction of *graph theory* would benefit from a different presentation of this problem.
- Another instructor combined problem 8 with the ideas from problems 3-5, which I think was excellent, though again the number of vertices involved should probably be kept to at most six.
- Problem 9 could benefit from a less intimidating presentation, or a discussion of whether undirected or directed graphs are more appropriate here.

[5] Counting the edges of K_6 is harder than you might expect.

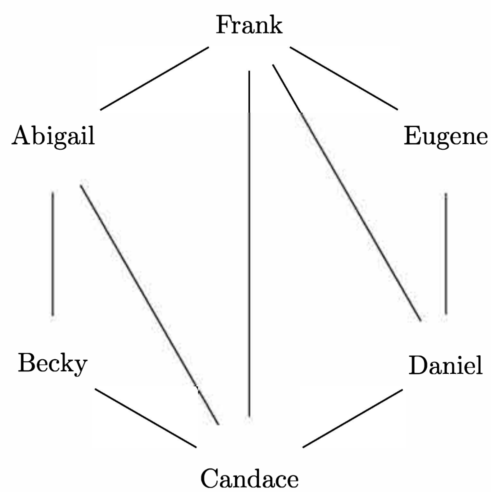
Graph Theory

4.1 Graphs and Digraphs

Graph theory is the study of networks.

This section uses a lot of color coding. You may want to ask your students if any of them have issues distinguishing between colors: you can easily change the color coding in the preamble to the .tex file for these notes, and give them a personalized copy.

Definition. *A **graph** is a collection of objects and relationships between pairs of those objects. We call the objects **vertices** (plural of **vertex**) and we call the relationships **edges**.*



In the last decade or so, online social networks have become very important, and these can be studied as graphs. For example, imagine Abigail, Becky, Candace, Daniel, Eugene, and Frank are all students at the same school - probably some of them are Facebook friends, and some are not. We can represent this information using a graph.

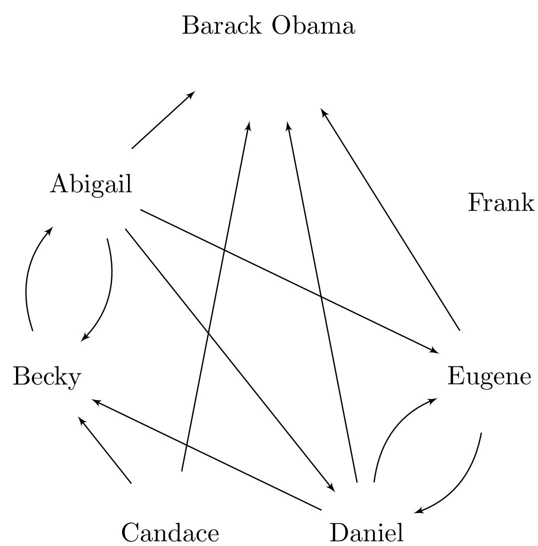
I suspect "Facebook friends" is rapidly becoming an archaic reference. Maybe use a different example in order to seem cool to the young people.

The vertices on this graph are the students. The edges are the lines between them, which represent that two students are Facebook friends. Abigail is Facebook friends with Becky, Candace, and Frank because there are edges between her vertex and their vertices. Abigail is not Facebook friends with Eugene or Daniel because there is no edge between her vertex and theirs.

Comprehension Check. *Who among the six students are Facebook friends with Becky? How many of the students are Facebook friends with Frank?*

Becky's friends are Candace and Abigail. Frank has four friends among the group.

Facebook friendship is a mutual or symmetric relationship - if Abigail is friends with Becky, then Becky must also be friends with Abigail. Not all relationships are symmetric, and we sometimes represent such relationships on graphs with the use of arrows. Keeping with the theme of social networks, Twitter users can choose to follow another user's messages. This is not a symmetric relationship - just because Daniel reads Barack Obama's tweets, does not mean Barack Obama returns the favor. Typically, high profile figures are followed by many people, but do not follow many people. We can use a graph with arrows to represent this type of relationship.



In this graph, the person at the tail of an arrow is a Twitter follower of the person at the head of the arrow. Candace has no followers. Barack Obama does not follow anyone. Some pairs of people (such as Abigail and Becky) do have a symmetric relationship - we choose to represent that with a pair of arrows, but there are other ways you could represent that symmetry. The drawing above is a typical representation of a directed graph or digraph.

Definition. *A **digraph** is a collection of **vertices** and **directed edges**, where a directed edge represents a one-way relationship between two vertices.*

Comprehension Check. *Which person in the digraph has the most followers? Who follows the most people? Which person appears not to know how to use Twitter? Which pair of people (besides Abigail and Becky) have a symmetric relationship?*

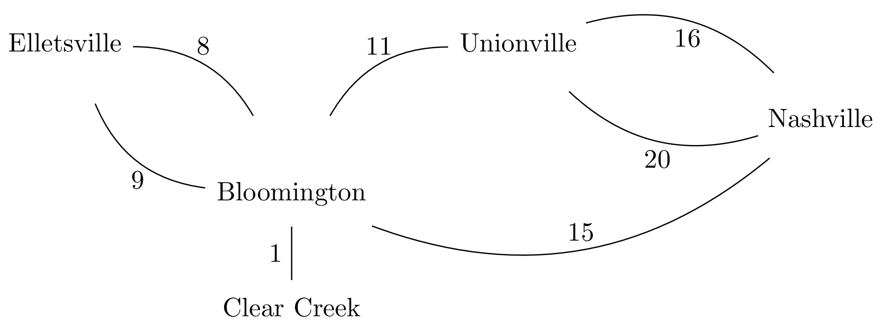
Barack Obama has the most followers. Abigail follows the most people. Frank is bad at Twitter. Daniel and Eugene have a mutual relationship.

Another aspect of a network we might want to represent is the magnitude of a relationship.

Definition. *When we wish to associate a number with each edge on a graph, we call that number the* ***weight*** *of that edge. We call each edge with a number a* ***weighted edge****, and we call a graph with weighted edges a* ***weighted graph****.*

A very typical use of weighted graphs is to represent connections and distances between places. For example, the following graph depicts a group of towns in Indiana, and the distances between them by road.

The number we associate with each edge often indicates the *significance* of that edge. Usually a big number means more significance, but it may be worth pointing out that here a larger number signifies a weaker relationship between the vertices.



According to this graph, the distance from Bloomington to Unionville is 11 miles. We do not typically write units on our graph because it clutters the diagram, but it is important to understand what the units are in context. There is no direct path from Unionville to Clear Creek, but the distance between those two cities is 12 miles via Bloomington.

Comprehension Check. *What is the shortest route from Nashville to Elletsville?*

The shortest route is 15 miles to Bloomington and then 8 miles to Elletsville for a total of 23 miles.

Observe that on this graph, there are multiple edges between some pairs of vertices.

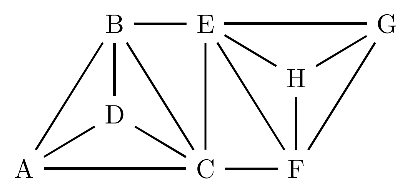
Definition. *We call a graph with multiple edges between some vertices a* ***multigraph****. Usually in this class we will consider multigraphs to be a special type of graph. In some texts, they are considered a similar but distinct object from graphs.*

Depending on a person's interests in the graph, they might choose to represent it with no duplicated edges. For example, a person who wanted to tour all five of these towns would always take the fastest route, and could safely ignore the redundant roads. They might redraw this graph without certain edges. On the other hand, a person who was interested in the roads themselves (perhaps a snow plow owner) would want to use the entire multigraph.

Exercises

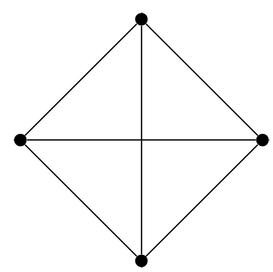
The **degree** of a vertex is the number of edges connected to that vertex.

1. Answer the following questions about this graph:



   (a) How many vertices does this graph have?
   (b) How many edges does this graph have?
   (c) What is the degree of vertex A?
   (d) What is the highest degree of any vertex on the graph?

2. Answer the following questions about this graph:



   (a) How many vertices does this graph have?
   (b) How many edges does this graph have?
   (c) What is the highest degree of any vertex on the graph?

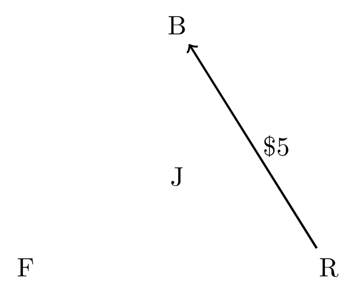
3. Alfred, Annie, Alex, Bob, Brittany, and Berthe are all attending a party. Each of the people whose names start with A already know each other, but know none of the people whose names start with B. The people with names that start with B all know each other already. Draw a graph representing each person as a vertex, and use edges to indicate that two people know each other. How many edges are there?

4. Using the same scenario as in the previous problem, draw a graph where edges represent that two people do not know each other. How many edges are there?

5. On a graph with 6 vertices, where there is an edge between every two vertices, how many edges are there? How does this relate to the previous two questions?

6. Freddie, Brian, Roger, and John all play in a band together. John also plays in a band with Jimmie and Robert. In fact, John plays in another band with Paul, George, and Ringo. Draw a graph in which each of these people is a vertex, and draw an edge between each pair of people who are in a band together.

7. Freddie, Brian, Roger, and John all go out for a night on the town: they agree that the following day, they will pay each other back for the costs of the evening. Freddie drives, and spends \$12 on gas. Brian pays \$60 for the group's dinner. Roger buys movie tickets, spending \$40. Use a *directed, weighted* graph to represent how much money each friend owes to each other. The first edge is already drawn: the weight is the amount that Roger owes Brian, minus the amount that Brian owes Roger.



8. Draw a graph where each vertex represents a state in New England, and edges represent that two states share a border. You may find it is helpful to draw your graph on top of this map of New England.

9. Lazyname Airlines offers daily flights between 6 major cities: New York City, Indianapolis, Atlanta, Reykjavik, Paris, and London. The table of available flights is given here.

Flight	**Departs**	**Arrives**	Cost
103	Indianapolis	Atlanta	$ 200
102	Indianapolis	New York	$ 400
201	New York City	Indianapolis	$ 400
203	New York City	Atlanta	$ 200
204	New York City	London	$ 800
301	Atlanta	Indianapolis	$ 200
302	Atlanta	New York	$ 200
304	Atlanta	London	$ 900
306	Atlanta	Reykjovik	$ 800
402	London	New York	$ 800
403	London	Atlanta	$ 900
405	London	Paris	$ 100
406	London	Reykjovik	$ 400
504	Paris	London	$ 100
506	Paris	Reykjovik	$ 600
603	Reykjovik	Atlanta	$ 800
604	Reykjovik	London	$ 400
605	Reykjovik	Paris	$ 600

Use a weighted graph to represent the cities Lazyname flies to, which flights are available, and how much each flight costs.

10. Every year Lazyname Airlines shuts down most of their flights for a week in celebration of President's Day, leaving only the following schedule.

Flight	**Departs**	**Arrives**	Cost
103	Indianapolis	Atlanta	$ 200
201	New York City	Indianapolis	$ 400
204	New York City	London	$ 800
302	Atlanta	New York	$ 200
406	London	Reykjovik	$ 400
504	Paris	London	$ 100
603	Reykjovik	Atlanta	$ 800
605	Reykjovik	Paris	$ 600

Draw a weighted digraph to represent the flights that are available. Is it possible to travel from any city to any other city using just these flights?

Decision and Beauty Assignment with Student Work

The following question from my *Voting Theory* course notes is the basis for a group activity.

> 11 Design a voting system which you feel fairly and accurately reflects the wishes of an electorate. You should be able to answer the following questions about your voting system, and they might give you some ideas about approaching this exercise:
>
> (a) What information does each voter put on their allot?
>
> (b) Are all voters equal? If not, how does the role of different voters vary?
>
> (c) Are all candidates equal? If not, how does the role of different candidates vary?
>
> (d) Is your system deterministic?
>
> (e) Is your system a Condorcet method?
>
> (f) Is your system similar to any of the systems discussed so far in class? If so, how have you improved it?
>
> Write down the rules of your system clearly and precisely, and answer the above questions about it.

Over the years I have used this activity with different prompting questions, so there is some variation in the solutions that I have documented on the following pages. I typically tell students they will be graded on their ability to describe the rules of their voting system unambiguously; I make this is an exercise about communication as much as about voting systems.

I give students whose descriptions are ambiguous feedback and the opportunity to resubmit. Usually these students resubmit the assignment individually, for logistical reasons. Also for logistical reasons, it was easier for me to get permission from those individual students than from groups for me to share their work, so the work that follows represents students' second attempt to describe their voting system.

I follow up on this activity later in the unit, by asking students to determine which fairness principles apply to their voting system. I have sometimes also discussed with students how strategic voting might be used in their voting system.

The voter profile would be evaluated by comparison between each 2 candidates, and the winner would be determined by the candidate with the greatest winning margin.

30	4	3	27	27	5
A	A	B	B	C	C
B	C	A	C	A	B
C	B	C	A	B	A

We would evaluate the candidates by comparing A+B, A+C, etc. individually.

61 voters support A over B; 35 support B over A.
Margin of $A > B = 26$
(etc.)
This process would be repeated comparing all pairs of candidates (x, y).

greenroom®

11. 10 points to distribute (per voter)
(3 candidates) - or any number less than 10 candidates

Ballot

Candidate X	- 9	5	0
y	- 1	3	10
z	- 0	2	0

each voter can give ~~the~~ each candidate their chosen number, giving them more power to give weight to each candidate vote than for instance, the Borda count, where the order decides the weight definitively. All voters and candidates are equal and the result is most likely definitive.

VOTING Homework 4 (VOTING SYSTEM DESIGN)

Rules:

11. a. Each voter casts their ballot in a social ordering.
    ex. A>B>C?

    b. All voters are equal, but their lowest ranked social ordering holds no weight.

    c. All candidates are equal.

    d. not deterministic

    e. not condorcet

    f. this system is very similar to the Borda Count and runoff systems

Steps:

1. allocate points for each candidate using the Borda Count.
2. have a runoff between the candidates with the two highest point values.

Ex:

	5	1	3	5	5	1
2	A	A	B	B	C	C
1	B	C	A	C	A	B
0	C	B	C	A	B	A

→ A: 20, B: 22, C: 18 → runoff - A: 11, B: 9

↓

A wins

#11

New voting system:

Plurality with a second and third place Borta count. Once the winner is chosen with plurality, they are taken out and a Borta count is done with second, third, fourth, ect candidates. This shows if the original plurality was correct or not.

example:

	10	10	9	6	4	15
2	~~A~~	~~A~~	B	B	C	C
1	B	C	~~A~~	C	~~A~~	B
0	C	B	C	~~A~~	B	~~A~~

A: 20
B: 15
C: 19

with plurality, A wins, then B and C go head to head in a Borta count and in this count A or the winner is not counted.

B: 30 + 25 = 55 votes
C: 38 + 16 = 54 votes

From this, we see that with plurality the result was A > C > B, but then when a Borta count is done between 2nd and 3rd place, the results will not always be the same, now in the borta count, B > C.

Decision and Beauty Sample Exam

This test was given over the 2019 summer semester during the graph theory unit. In reflection:

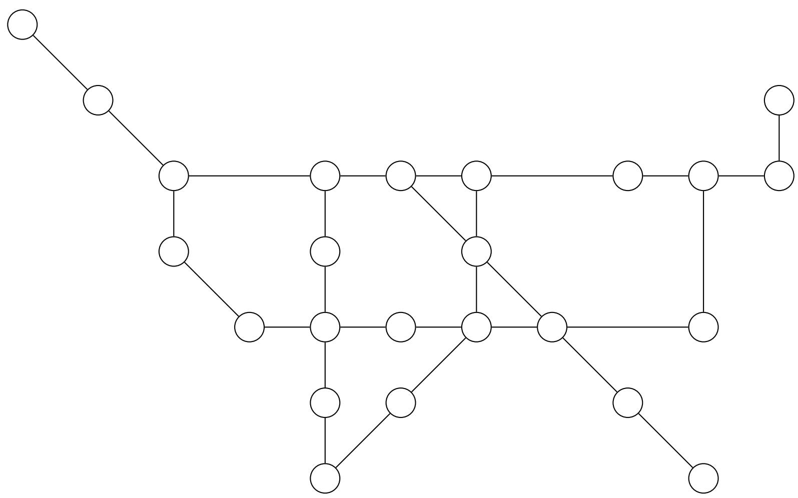
- Problem 1(a) is harder than I anticipated when I wrote the test; most students were able to produce an intelligent answer using sensible methods, but few found the optimal answer. The other instructor and I agreed on a generous grading scheme for this problem that reflected this, so that students who had used the methods taught in the class still earned nearly full credit. But I would not intentionally use a problem this technically difficult in the future.

- Problem 4 is technically easy (it involves a graph with 6 vertices and 8 edges) but conceptually a little challenging, and goes slightly beyond typical scheduling problems. In spite of not having been shown a problem exactly like this in the course, most students solved this problem correctly. I prefer problems of this kind, where students see something new on fair terms, to those like problem 1, where the challenge is mostly technical.

Name:

Read each question carefully. You should show all of your work in order to receive full credit on this exam. Believe in yourself, as I believe in you.

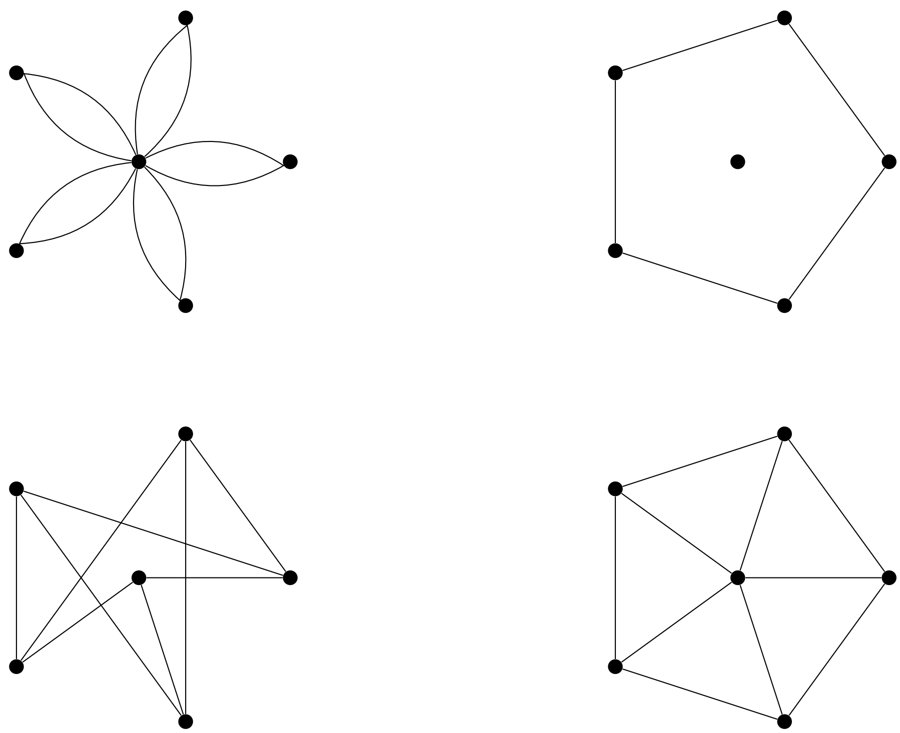
1. The graph below represents a railray map - each oval is a station, and each section of rail between stations is one mile.



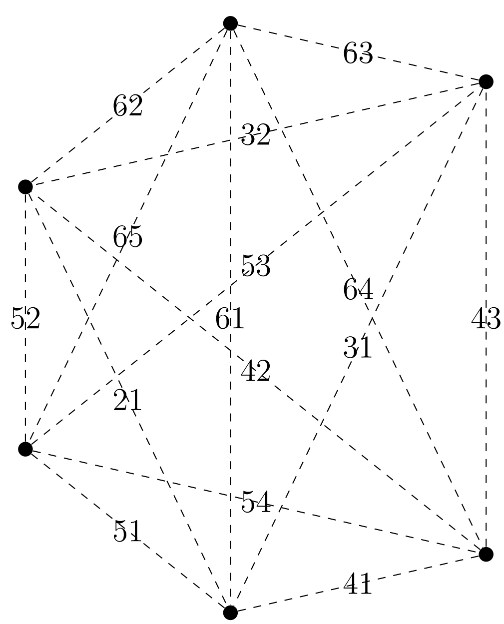
   (a) Mr. Miyazaki wishes to sightsee from the railway, and he doesn't want his trip to become repetitious. What is the longest route he can take without repeating any section of railway, if he doesn't mind revisiting some stations?

   (b) Dr. Goldman also wishes to sightsee, but she doesn't want to miss any part of the railway. She also wants to end her trip at the same station she starts at. What is the shortest route she can take to accomplish this?

2. Determine which of the graphs below have Hamiltonian circuits, and which do not. If a graph does not have a Hamiltonian circuit, you should explain why.



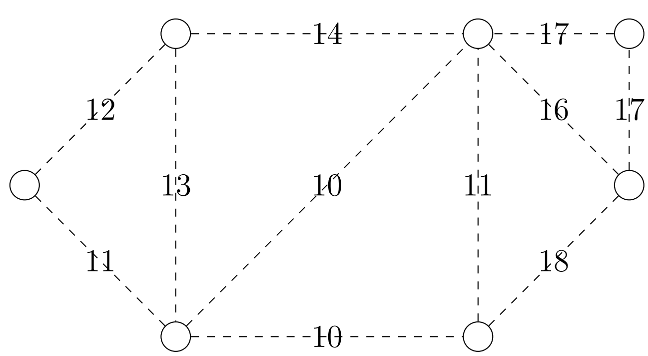
3. Use the sorted edges algorithm to find a Hamiltonian circuit on the graph below.



4. Amos, Ben, Cindy, Delmar, Escobar, and Francis are going rafting in three rafts, with two people in each raft. Amos and Ben are both too heavy to go in the same raft, whereas Escobar, Delmar, and Francis are all too light to be paired up. Cindy and Ben are both adventurous and plan to hit the most dangerous parts of the river; neither of them can share a raft with Amos or Delmar, who are both skittish. Cindy is allergic to Escobar's cat, which travels with him *everywhere*.

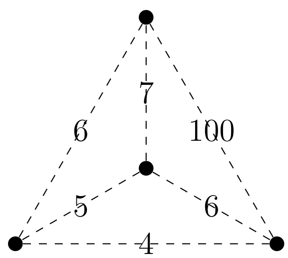
   Sketch a graph of the conflicts prohibiting these people from rafting together, find a coloring of that graph, and deduce how the rafters should be paired up.

5. The nodes in the graph below represent military bases that need to be connected into a communications network. The weighted edges of the graph represent the cost of establishing a connection (in thounds of dollars) between two bases. What is the minimum cost to build a <u>connected</u> network that includes every base?



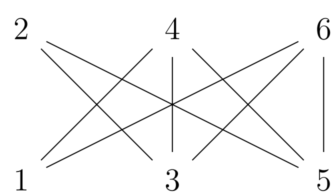
In each of the following problems, you are asked to apply an algorithm from the class. You will also be asked whether the algorithm gives an optimal solution to the problem and, if not, to produce a better solution.

6. (a) Use the Nearest Neighbor algorithm (starting from the middle vertex) to find a cheap Hamiltonian circuit on this graph, and say the cost of that circuit.



   (b) Was this the cheapest possible Hamiltonian circuit? If not, say the cost of the cheapest Hamiltonian circuit.

7. (a) Use the greedy coloring algorithm to color this graph.



   (b) Is this a minimal coloring? If not, what is the chromatic number of this graph?

Mathematics of Voting Syllabus

The following ten pages contain the syllabus for the voting theory course I taught at the Collins Living Learning Center at Indiana University, in the spring semester of 2022.

The Mathematics of Voting – CLLC 130 Section 10686, Spring 2022

Classroom and Time: FQ 012B, MW 1:45 PM–3:00 PM

Instructor: My name is Daniel Condon (call me Daniel). I am a math PhD student who has been interested in the mathematics of voting for several years, as a teacher, researcher, and voter.

Instructor Contact: My email address dmcondon@iu.edu, which I check often during ordinary business hours; this is generally the best way to contact me.

Office Hours: 2 hours/week at the Shea Coffeehouse, time TBD. No appointment necessary.

Course Description

Abstract: You and your fellow students play the role of a democratic activists in a fictional democracy. You will use what you learn in the class to report on political developments in the region and argue for or against changes to the rules of the democracy. You will also act as voters and will learn how political strategists can manipulate the rules of democracy to their advantage.

This is a course in the procedural, rather than social, aspects of elections. We will study the mathematics of such procedures as first-past-the-post voting, ranked-choice voting, approval voting, districting, and district apportionment, to see how the specific rules of a democracy impact different notions of 'fairness.' These and other examples will help us to see how rigid notions of fairness may contradict each other or be undermined.

Prerequisites: You do not need an advanced background in math or political science to participate. Our calculations will be limited to elementary arithmetic, and you are allowed a four-function calculator (which is all you will need). Some of the mathematical concepts may still be challenging.

Learning Outcomes: By the end of this course, students will:

- Apply common democratic procedures to determine election winners and apportion voting districts according to those procedures.
- Apply strategic voting methods to affect elections in their favor.
- Analyze notions of fairness and understand their relationships with each other and strategic voting methods.
- Evaluate common democratic procedures in terms of fairness criteria.
- Evaluate common arguments in the discourse on election reform.
- Create new democratic procedures and evaluate the ways in which they are fair/unfair, and how they may be susceptible to strategic manipulation.
- Communicate unambiguously about the rules of elections and fairness criteria.

Readings and Resources: the main text for this course will be *A Mathematical Look at Politics* by Robinson, Ullman, and it is recommended students obtain a copy of this text. The course will also use material from:

- *Chaotic Elections, a Mathematician Looks at Voting* by Donald G Saari. This text investigates strategic voting to a greater depth than the main text.

- *Social Choice and Individual Values*, by Kenneth Arrow. This seminal text in voting theory not only gives major results but also contextualizes them far more broadly than the main text.
- *Social Choice Theory and Research* by Paul E. Johnson. Johnson describes the so-called “Chaos Theorems” which demonstrate how sequenced referendums may cause a rational electorate to agree to nearly anything.
- *Votes from Seats* by Matthew Shugart, Rein Taagepera. Shugart and Taagepera describe a statistical model of the number of political parties that can arise in a country based on statistics about its culture. This is a radically different approach to our topic than the main text takes, but the results are interesting and relevant to our discussion of voting methods and will be presented in lecture.
- *Electoral Systems: A Comparative Introduction*, by David Farrell. Farrell describes methods with multiple winners in greater depth than the main text.
- *Honeybee Democracy* by Thomas Seeley. Seeley frames the behavior of bees as a procedural and democratic process, which demonstrates an unexpected application of the theory of voting.

Lecture notes will be provided for information not found in the main text.

Class Structure and Expectations

Students will form groups of 4-6 with whom they will complete most assignments throughout the class.

Topics will often be introduced through lecture or guided activities, typical of a math class. You will continue to learn about these topics through activities that require different kinds of engagement, including discussion, roleplay, and written reflection.

As part of an ongoing activity, students will play the role of political activists in an evolving democracy. You must use what you have learned to argue for and against changes to the rules of the democracy, and report on developments that may make their political system less fair. You will also vote on issues, and evaluate when and how political strategists may use the rules of the democracy to their advantage.

Students are expected to interact with their classmates and instructor in a manner in accordance with the IU Code of Student Rights, Responsibilities, and Conduct.

Technology Policy: You are welcome to use your laptops to take notes and participate in the class. Students using laptops for activities unrelated to the class will be asked not to use them in the future.

Academic Integrity: As a student at IU, you are expected to adhere to the standards and policies detailed in the Code of Student Rights, Responsibilities, and Conduct. When you submit an assignment with your name on it, you are signifying that the work contained therein is yours, unless otherwise cited or referenced. Any ideas or materials taken from another source for either written or oral use must be fully acknowledged. All suspected violations of the Code will be reported to the Dean of Students and handled according to University policies. If you are unsure about the expectations for completing an assignment or taking an exam, be sure to seek clarification beforehand. The following is the IU policy on plagiarism, which can be found here: http://www.indiana.edu/~istd/definition.html

3. Plagiarism: Plagiarism is defined as presenting someone else's work, including the work of other students, as one's own. Any ideas or materials taken from another source for either written or oral use must be fully acknowledged, unless the information is common knowledge. What is considered "common knowledge" may differ from course to course.

a. A student must not adopt or reproduce ideas, opinions, theories, formulas, graphics, or pictures of another person without acknowledgment.
b. A student must give credit to the originality of others and acknowledge an indebtedness whenever:
1. Directly quoting another person's actual words, whether oral or written;
2. Using another person's ideas, opinions, or theories;
3. Paraphrasing the words, ideas, opinions, or theories of others, whether oral or written;
4. Borrowing facts, statistics, or illustrative material; or
5. Offering materials assembled or collected by others in the form of projects or collections without acknowledgment.

Grading

60% of your grade comes from **assignments**. The assignments each day are weighted equally by day. Math problems will be graded for accuracy, and you are expected to show your work. Non-math problems (usually asking you to comment on a democratic procedure or reflect on a discussion or activity) may include very specific prompts, in which case your grade will reflect whether you addressed those prompts and (if applicable) whether your response was precise and mathematically accurate. These activities may also be open ended and graded for accuracy and completion. Assignments will typically be due at the end of class, or at the start of the following class.

10% of your grade comes from the **exam**, which will be graded for accuracy.

10% of your grade comes from each of two **projects** at the end of the course. In the last week, your group may give a presentation on one of those projects to earn some extra points on that project.

10% of your grade comes from **attendance**. You must attend 70% of classes to pass the class. If you attend 90% of classes or more, you will receive full credit on your attendance grade. If you attend between 70% and 90% of classes, your grade will be interpolated linearly (for example, attending 80% of classes will earn you a 50% attendance grade). If you miss many classes due to extreme circumstances, the attendance policy may be altered to your benefit at the instructor's discretion.

Dropped Assignments: 10% of assignment grades will be dropped.

Absences: if being absent from class makes it impossible for you to complete an assignment, you will automatically score a 0 on that assignment. The instructor will try to accommodate you if it is feasible for you to complete an assignment for a day you missed. As with the attendance policy, if you miss many classes due to extreme circumstances this policy may be altered to your benefit at the instructor's discretion.

Late Work: late work will never be accepted after solutions are posted. Work where this does not apply (your personal reflection on an activity) may be accepted late at the instructor's discretion.

Resources and Services

Bias reporting: Every member of our classroom has a right to participate in the course without being subject to harassment or discrimination based on age, color, religion, disability (physical or mental), race, ethnicity, national origin, sex, gender, gender identity, sexual orientation, marital status, or veteran status. To learn more, or to report an incident of bias, use the link below:

https://studentaffairs.indiana.edu/student-support/get-help/report-bias-incident/index.html

CAPS: IU offers Counseling and Psychological Services to students. The services are confidential, and your first appointment is free.

https://healthcenter.indiana.edu/counseling/

Disability services: Every attempt will be made to accommodate qualified students with disabilities (e.g. mental health, learning, chronic health, physical, hearing, vision, neurological, etc.). You must have established your eligibility for support services through the appropriate once that services students with disabilities. Note that services are confidential, may take time to put into place and are not retroactive; captions and alternate media for print materials may take three or more weeks to get produced. Please contact Disability Services for Students at

http://disabilityservices.indiana.edu

or 812-855-7578 as soon as possible if accommodations are needed. The office is located on the third floor, west tower, of the Wells Library, Room W302. Walk-ins are welcome 8AM to 5PM, Monday through Friday. You can also locate a variety of campus resources for students and visitors that need assistance at:

http://www.iu.edu/~ada/index.shtml.

Religious observances: Students with conflicts between course requirements (e.g. exams) and religious observances must contact their instructors during the first week of the term and follow the procedures outlined by campus policy, available at:

http://enrollmentbulletin.indiana.edu/pages/relo.php

Sexual misconduct: As your instructors, one of our responsibilities is to help create a safe learning environment on our campus. Title IX and our own Sexual Misconduct policy prohibit sexual misconduct. If you have experienced sexual misconduct, or know someone who has, the University can help. If you are seeking help and would like to speak to someone confidentially, support resources for individuals who have experienced sexual assault are available 24 hours a day. Call (812) 855-8900. More information about available resources can be found here:

http://stopsexualviolence.iu.edu/help/index.html.

It is also important that you know that federal regulations and University policy require us to promptly convey any information about potential sexual misconduct known to us to our campus' Deputy Title IX Coordinator or IU's Title IX Coordinator. In that event, they will work with a small number of others on campus to ensure that appropriate measures are taken and resources are made available to the student who may have been harmed. Protecting a student's privacy is of utmost concern, and all involved will only share information with those that need to know to ensure the University can respond and assist. To learn more, we encourage you to visit

http://stopsexualviolence.iu.edu.

Course Calendar

The calendar below lists topics for each week, assignments to be turned in, readings, and additional activities.

Week 1

Syllabus and class expectations, introduction to Social Choice, the vocabulary of voting theory, the relationship between democracy and voting, First-Past-the-Post, the spoiler effect, Duverger's law, Game Theory

Assignments:

- Reflection on the relationship between democracy and voting
- Propose aesthetics to vote on
- First-Past-the-Post and spoiler effect worksheet
- Reflection on Duverger's law

Additional Activities:

- Choosing groups equitably
- Voting roleplay
- Duverger's law roleplay
- Game theory crash course

Reading

AMLAT [A Mathematical Look at Politics] Sections 1.0, 1.1.

The game theory topics are covered in AMLAT Sections 13.0, 13.1, 13.2, 13.3, 15.1.

Notes on the Spoiler Effect and Duverger's Law will be provided.

Students will be asked to read about a problem from social choice.

Week 2

Strategic voting, Ranked Choice Voting, runoffs, pushover voting

Assignments:

- Reflection on strategic voting and "rationality"
- Ranked Choice Voting worksheet
- Reflection on Ranked Choice Voting

Additional Activities:

- Voting roleplay

Reading

Ranked Choice Voting is described in AMLAT as Hare's method, pg 32-34.

Notes on pushover voting will be provided.

Week 3

The Borda count, burying, dark horse candidates, positional methods, cardinal voting, approval voting, score voting, bullet voting

Assignments:

- The Borda count and burying worksheet
- Dark horse reflection
- Cardinal voting worksheet
- Voting stakes reflection

Additional Activities

- Voting roleplay
- Linearity of scores investigation

Reading

The Borda count is discussed in AMLAT pg 30-32.

The positional method is discussed in AMLAT on pg 37.

Notes on cardinal voting methods will be provided.

Week 4

Condorcet criteria, the Condorcet paradox, Copeland's method, Dodgeson's method, Baldwin's method, agenda voting, and the Chaos Theorems

Assignments:

- Condorcet methods worksheet
- Complexity reflection
- Agenda voting worksheet
- Neutrality reflection

Additional Activities:

- Implementing the Condorcet criteria
- Bracket manipulation activity

Reading

Copeland's method and agenda voting are discussed in AMLAT, pg 35-37.

The Condorcet criteria are discussed in AMLAT, pg 51-53.

The Condorcet paradox is discussed in AMLAT, Section 5.1.

Notes will be provided on Dodgeson's and Baldwin's methods, as well as the Chaos Theorems.

Week 5

Proportional voting methods, Single Transferrable Vote with varying quotas, list systems, wasted votes, multi-party systems, Votes from Seats

Assignments:

- Single Transferrable Vote worksheet
- List systems worksheet
- Multi-party systems reflection

Additional Activities:

- Voting roleplay

Notes will be provided on voting systems with multiple winners. These are also discussed briefly in AMLAT pg 122-123.

Week 6

No new material, just assignments

Assignments:

- Design a voting method
- Prepare a short presentation on a voting method, meant for public education

Reading

This is a good opportunity to revisit Chapters 2 and 6 of AMLAT.

Week 7

Notions of fairness and democracy, fairness principles, intro to proofs, the relationship between fairness principles and strategic voting

Assignments:

- Reflection on notions of fairness and democracy
- Fairness principles and strategy worksheet

Additional Activities:

- Evaluate student voting methods
- Introduction to proofs

Reading

Basic notions of fairness are discussed in Chapter 3 of AMLAT.

The relationships between fairness principles and certain specific voting methods are discussed in Chapter 4 of AMLAT.

Brief notes regarding the relationship between fairness principles and strategic voting will be provided.

Week 8

May's Theorem, toy voting methods, Arrow's Theorem, Gibbard-Satterthwaite Theorems

Assignments:

- Arrow's Theorem worksheet
- Impossibility Theorems reflection

Additional Activities:

- Chart which voting methods satisfy which fairness principles
- Revisit student-designed voting methods

Reading

The relationships between fairness principles and certain specific voting methods are discussed in Chapter 4 of AMLAT.

Arrow's Theorem is stated in section 5.2 of AMLAT, and its proof follows throughout Chapter 5.

Notes will be given on the Gibbard-Satterthwaite Theorems.

Week 9

Apportioning districts, vocabulary of apportionment, Hamilton's method, fairness in apportionment, quota systems, house monotonicity and the Alabama paradox, population monotonicity, the new states paradox

Assignments:

- Apportionment problem reflection
- Hamilton's method worksheet
- Fairness in apportionment reflection

Additional Activity:

- Invent a district apportionment procedure, without quota violation

Reading

All of these topics are covered in Chapter 7 of AMLAT.

Spring Break

Week 10

District apportionment impossibility theorems, Jefferson's method, Balinski-Young method, Hill's method, Webster's method, history of district apportionment in America

Assignments:

- Apportionment methods worksheet
- Fairness principles worksheet
- Reflection on impossibility theorems

Reading

Jefferson's method is discussed in 8.1-8.3 of AMLAT.

Webster's method is discussed on pg 157 of AMLAT.

Hill's method is discussed in AMLAT, pg 158-159.

Impossibility theorems are discussed in 9.6 of AMLAT.

Balinski and Young's method is discussed in 10.1 of AMLAT.

History of district apportionment in America is discussed in Notes on Part II *of AMLAT.*

Week 11

Districting, history of districting and gerrymandering in America, shortest split-line, geometric districting methods

Assignments:

- Districting discussion reflection
- Geometric methods worksheet

Additional Activity

- Municipal districting activity
- Districting roleplay

Reading

Notes will be provided on gerrymandering, districting, and geometric methods of districting.

Week 12

Gerrymandering, the efficiency gap and other geometric tests

Assignment:

- Gerrymandering reflection
- Gerrymandering tests worksheet
- Discourse around gerrymandering tests reflection

Additional Activity

- Gerrymandering roleplay

Reading

Notes will be provided on gerrymandering, gerrymandering tests, and the discourse around these.

Week 13

This week we review everything we have learned, and we have a math test. The test is graded for accuracy, but students will be able to turn in a corrected test the following week to earn back up to half the points they lost.

Week 14

Parliamentary and presidential systems, a statistical model of multi-party systems, the Electoral College, national election systems

Assignments:

- Electoral College reflection

Additional Activity

- Design a national election system.

Week 15

Mini talks: honeybee democracy, psychology of ballot presentation, ensemble learning, voting reform, sortition, Collins's student government

Assignments:

- Students prepare an informational project about one of these topics.

Week 16

Students have the chance to polish one of their projects from the previous two weeks, and then present it. This is an opportunity to increase their grade for that project.

Discrete Math Syllabus

The following 8 pages are the course syllabus for Discrete Math, which I am teaching the fall semester of 2022. Certain personal information has been redacted.

Since I am likely to teach this course again in the spring, I am considering the following changes:

- List letter grade cutoffs on the syllabus, instead of presenting that information elsewhere
- Be more explicit about academic integrity, and make it its own document. Some students had difficulty interpreting why some of the examples of inappropriate behavior were inappropriate. When I asked students about their interpretations of the examples, most took an extremely hard line against cheating and proposed policies that would have made collaboration virtually impossible.

Math 200 - Discrete Math

Fall 2022

Classroom: Wachenheim 113
Section 01 class time: Tuesday, Thursday from 8:30 am - 9:45 am
Section 02 class time: Tuesday, Thursday from 9:55 am - 11:10 am

Instructor name: Daniel Condon
Instructor email: dc25@williams.edu
Instructor office: Wachenheim 335
Office hours: Tuesday 2 pm - 4 pm, Friday 9 am - 11 am, and by appointment

TAs: Names redacted

Evening help sessions:			
	Monday	8-9	Room redacted
	Tuesday	8-9	Room redacted
	Wednesday	7-8	Room redacted
	Thursday	9-10	Room redacted

Textbook: *Discrete Mathematics with Applications*, by Susanna Epp, 5th Edition
Course Webste: GLOW (accessible at glow.williams.edu, or through the canvas app)

Course Description

Discrete Math includes topics which are important or even foundational to various mathematical subjects, including theoretical computer science, logic, finite group theory, number theory, probability, graph theory, and combinatorics. The topics covered are primarily "discrete" as opposed to "continuous." For example:

- If one studies the properties of the integers, without appealing to the existence of other numbers, one is practicing discrete math rather than continuous math. See also:
- Finite systems, like a computer network, a road map, or a finite algorithm;
- Systems with finitely many arrangements, like a Rubik's cube, a cypher, or a deck of cards;
- Locally finite systems, like a board game that doesn't necessarily end, a tiling of the plane, or an algorithm with infinitely many possible inputs.

Discrete math classes are often required for math majors and computer science majors. For many students, discrete math is the first time they are expected to write proofs at a college level. It may also be the first class where the emphasis shifts toward understanding objects, rather than learning algorithms. These changes represent an escalation in abstraction, so many students find discrete math to be a very challenging class; however, it can also be extremely rewarding.

Course Policies

Class Participation

You should attend every class, and participate actively: ask questions, complete all the homework, and reach out for help when you need it. This is a very challenging course, and it is easy to fall behind, especially if you do not participate fully. If you are going to miss class, please let me know as soon as possible.

Sickness Policy

If you are sick, please stay home. If homework is due on the day you would miss, I will ask you to submit your work as a pdf on GLOW. If you desire an extension on homework due to an illness, please email me to discuss your options.

Workload Statement

For every hour in class, you should plan to spend 3 hours outside of class studying or completing assignments. If you are finding yourself spending considerably more time than that on the course, please reach out to me.

Classroom Policies

In this course, you will be expected to participate in group and class discussions. You are expected to treat others with respect, and you deserve to be treated with respect. Our class will have a community learning agreement to guide these interactions.

Inclusion Statement

The Williams community embraces diversity of age, background, beliefs, ethnicity, gender, gender identity, gender expression, national origin, religious affiliation, sexual orientation, and other visible and non visible categories. I welcome all students in this course and expect that all students contribute to a respectful, welcoming and inclusive environment. If you have any concerns about classroom climate, please come to me to share your concern.

Pronoun Policy

In this class, we use the name and pronouns that individuals ask us to use as a sign of mutual respect. I will use the pronouns you have indicated on GLOW unless you alert me to a different pronoun. When talking or referring to someone in the class whose pronouns you do not know, you can use the second-person pronoun "you." That said, everyone makes mistakes—in general, should you use an incorrect pronoun or name, the best course of action is to make a quick correction and move on, rather than dwelling on it.

Assignments

Homework

You are expected to work on homework with other students. Most real-world mathematics is a collaborative exercise, and working with others is good for you. However, your writeup should be your own work. You should also acknowledge people you worked with, in writing, when you submit your work. For example:

Appropriate	Inappropriate
Alice and Bob have a discussion about a problem after class. They each go to their separate homes and write up a solution.	Alice and Bob have a google doc where they write up solutions to homework. They both submit these solutions, with a few changes.
Alice and Bob decide to work on a homework set in the same room. They each get stumped on a couple problems, which the other is able to explain.	Alice and Bob decide to work on a homework set in the same room. Alice solves the odd problems, Bob solves the even problems, and then they share solutions.
Alice helps Bob by reading a proof that he wrote; she suggests a couple of changes that might make it easier to read.	Alice helps Bob by reading a proof that he wrote; it's pretty bad, so she gives him her own work as an example.
Alice is stumped by a problem, so she asks Bob for help. Bob shows her a standard technique, which he says was covered in a class she missed. Alice then completes the problem herself, and does not acknowledge Bob's help when she submits her homework.	Alice is stumped by a problem, so she asks Bob for help. Bob explains his solution to the problem, which Alice writes up. Alice does not acknowledge Bob's help when she submits her homework.

In the first three appropriate situations in the table, Alice and Bob should write on their homework the name of the person they collaborated with. Please note that inappropriate behavior is mostly plagiarism, and actions that enable plagiarism. How to respond to inappropriate behavior will be part of our community learning agreement.

Legibility
If your homework/test is not legible, it cannot be graded, and you risk receiving a zero. You should also organize your work in a way that is easy to read and understand; this may cause you to use slightly more paper.

Late Homework
Homework is due at the beginning of class. Homework may be accepted late at the instructor's discretion, however it will never be accepted after solutions have been published.

Dropped Assignments
Your lowest-scoring homework assignment will be dropped from your grade. Additional assignment grades may be dropped at the instructor's discretion.

Quizzes
In this class, a quiz is a short assignment to be completed outside of class, without working with others. On these, you are expected to write a proof. You will have two days to turn in your work, after which you may receive feedback and be asked to make corrections to your proof before receiving a grade. If the instructor has not prepared a quiz, a grade representing attendance may be recorded instead.

Take-Home Tests
This class has two take-home tests. For these, you are permitted to use your notes or textbook or other resources available to you, but you are expected to work alone and not to solicit help. Dates for these tests are listed below.

In-Class Tests
This class has a midterm which is a written tests taken in the classroom. You must complete these without the aid of a calculator, and you will not have access to notes, the textbook, or other resources. Dates for these tests are listed below.

Final Exam
Your final exam will also be in-class.

Curves
You should not expect assignment or test grades to be curved; I do not expect a curve to be necessary. If a curve is implemented, it will: not cause anyone's letter grade to decrease; preserve the order of grades.

Grades
Your grade is a weighted average of your grades across categories. For categories with more than one assignment, all assignments contribute equal weight (except any which are dropped).

Homework	25%
Quizzes	15%
Test 1, take-home, due Oct 6	10%
Midterm, in-class, on Oct 20	15%
Test 3, take-home, due Nov 15	10%
Final, in-class, date Dec 19	25%

Resources

Office Hours
For every college class you take, you should probably attend your instructors' office hours at least once. Most instructors will appreciate it even if you only come to introduce yourself. Meeting you makes it easier for us to know who you are, and give you more meaningful feedback on assignments. It is also normal to attend office hours for extra help, whether that is with understanding the class material or going beyond it.

Help Sessions
Your TAs will be leading help sessions. You will have the opportunity to ask questions about the class or homework, and you are welcome to attend simply to work on homework. These are an excellent resource most students should take advantage of, at least some of the time.

Peer Tutoring
Peer tutoring resources are listed on GLOW.

Health/Accessibility Resources
Students with disabilities or disabling conditions who experience barriers in this course are encouraged to contact me to discuss options for access and full course participation. The Office of Accessible Education is also available to facilitate the removal of barriers and to ensure access and reasonable accommodations. Students with documented disabilities or disabling conditions of any kind who may need accommodations for this course or who have questions about appropriate resources are encouraged to contact the Office of Accessible Education at oaestaff@williams.edu. See also:
https://www.williams.edu/accessible-education/academic-accommodations/
https://www.williams.edu/accessible-education/applying-for-accommodations/

Information on the Williams College Honor Code
https://sites.williams.edu/honor-system/
https://dean.williams.edu/academic-misconduct-honor-code/

Some succinct advice (from communication with Gretchen Long, Dean of the College):

> *"Students can exchange broad ideas or general approaches toward problem sets with other students, but may not engage in any joint writing or step-by-step problem solving. One way to be sure you are not violating the honor code is to refrain from writing/typing/crafting your response to the assignment with others. Rather, save the writing until you are on your own and working independently."*

Health Resources, Including Mental Health
https://health.williams.edu/mental-health-assessment-and-treatment/
https://health.williams.edu/articles/welcome-to-new-students/

Title IX
The following is taken from https://titleix.williams.edu/, which also links to more resources.

> *Title IX is a federal civil rights law passed as part of the Education Amendments of 1972. This law protects people from discrimination based on sex in education programs or activities that receive Federal financial assistance. Title IX states that: "No person in the United States shall, on the basis of sex, be excluded from participation in, be denied the benefits of, or be subjected to discrimination under any education program or activity receiving Federal financial assistance." Title IX applies to any institution receiving federal financial assistance from the Department of Education, including state and local educational agencies. Educational programs and activities that receive federal funds from the Department of Education must operate in a nondiscriminatory manner. Also, a recipient may not retaliate against any person for opposing an unlawful educational practice or policy, or because a person made charges, testified, or participated in any complaint action under Title IX.*

Sections and Assignments by Date

Date	Sections	Assignment Due	Notes
Thr, Sep-08	1.1, 1.2, 1.3, 1.4		
Tue, Sep-13	2.1, 2.2, 2.3		
Thr, Sep-15	3.1, 3.2, 3.3, 3.4	HW 1 Due	
Tue, Sep-20	4.1, 4.2, 4.3		
Thr, Sep-22	4.4, 4.5, 4.6, 4.10	HW 2 Due	Take-home is released
Tue, Sep-27	4.7, 4.8, 4.9		
Thr, Sep-29	4.5, 8.4	Test 1 Due	
Tue, Oct-04	5.1, 5.2	HW 3 Due	Short
Thr, Oct-06	5.2, 5.3, 5.4		
Tue, Oct 11	NO CLASS		
Thr, Oct-13	5.6, 5.7, 5.8	HW 4 Due	
Tue, Oct-18	Review	HW 5 Due	Short
Thr, Oct-20	MIDTERM		
Tue, Oct-25	6.1, 6.2, 6.3, 6.4		
Thr, Oct-27	7.1, 7.2, 7.3		
Tue, Nov-01	7.4	HW 6 Due	
Thr, Nov-03	8.1, 8.2, 8.3		
Tue, Nov-08	9.1, 9.2	HW 7 Due	Take-home is released
Thr, Nov-10	9.3, 9.5		
Tue, Nov-15	9.5, 9.6, 9.7	Test 3 Due	
Thr, Nov-17	9.8, 9.9		
Tue, Nov-22	9.4	HW 8 Due	Long
Thr, Nov 24	NO CLASS		
Tue, Nov-29	10.1, 10.2	HW 9 Due	Short
Thr, Dec-01	10.2, 10.3		
Tue, Dec-06	10.4, 10.5, 10.6		
Thr, Dec-08	Review	HW 10 Due	Long

Community Learning Agreement

We are here to learn together. To ensure that our time together is enriching, students and faculty will abide by the terms of this agreement. Anyone in our intellectual community can suggest an addition; the group will decide to accept, reject, or revise it.

1. **Hang in There.** We are all here to learn, and most of our learning will take place outside of our short time together inside the classroom each week. Remaining enrolled in this course means that you are ready to devote the time, effort, and energy that learning about this challenging, but rewarding, topic deserves.

2. **Take Care of Yourself.** The challenges of this class may represent a significant intellectual hurdle, and possibly also a significant emotional hurdle. We in this community will prepare for growing pains as we overcome those challenges, and practice self-care and self-respect along the way.

3. **Be Respectful of Others.** We agree not to interrupt one another, because we all learn at different speeds and communicate in different ways. If someone gets excited or impatient with another member of the learning community and interrupts or shows frustration at another person's learning process, we each have a responsibility to the community to gently but firmly remind each other of this agreement.

4. **Work Collaboratively.** When we are working together in groups, we each agree to communicate and respond in a timely and respectful manner. Something that feels "easy" to you may be very difficult for someone else; remember that something else that is difficult for you may feel very "easy" for that same person. We learn best when we approach learning as a collaborative rather than a competitive activity.

5. **Act with Integrity.** In this community, we do not misrepresent our work by committing plagiarism, or enabling plagiarism.

   (a) When we solve problems on assignments using resources outside of the class, we cite those sources.

   (b) When we solve problems with other people, we acknowledge their contributions in writing.

   (c) When we work with other people, we strive to contribute to and understand each part of an assignment. We do not assign different parts of assignments to different members of a group.

   (d) When we work with other people, we ensure our ownership of our work by writing our solutions separately. It is not appropriate to write solutions using a template created by another student, or created in a group.

   (e) It is appropriate to solicit or provide feedback on a completed proof, however it is not appropriate to provide a completed proof to another student to use as an example.

   (f) When another student asks us to act in a way we believe violates this agreement, we explain that belief to them.

   (g) If serious or repeated violations of this agreement occur, it is appropriate to consult with the instructor.

Syllabus Addendum

I wanted a one page mini-syllabus on which to outline policies that are subject to change, or didn't fit thematically with any of the more formal sections.

Classroom Computer Policy

It is extremely difficult to type math notes at a college level; most math is typeset using a coding system called LaTeX, which has a steep learning curve. Because laptops can be used for many distracting things, but will probably not be useful for participating in the class, I will ask you not to use them in class. You may take written notes on a tablet if that is your preference, and you may ask for an exception if you feel this policy limits how you would like to engage with the class.

Mask Policy

For the time being, masking is optional in this class.

My Name

I invite you to call me "Daniel," which is my first name. If you are more comfortable calling me "Professor Condon" or "Doctor Condon" that is OK.

Email Policy

I don't feel strongly about formality in emails, but a greeting, body, and sign-off are appreciated in the first email in a thread.

Comment Box

If you ever want to tell me something anonymously, you can submit a comment using this google form:

Link redacted

You can indicate your section if you feel that is relevant. You can also ask a question anonymously; if appropriate, I can answer the question in class.

GroupMe

There is a GroupMe chat for this class, made by the TAs. I am not part of the chat. You can join it at this link:

Link redacted

Discrete Math Midterm

The following three pages contain a midterm I gave to the discrete math class I am teaching in Fall of 2022. The test has been formatted to fit the questions on one page, but each question was on a separate page in the version given to students.

Because writing and problem solving have been ongoing themes in this class that are more important than any particular topic, I designed the test to deemphasize individual topics by allowing students to skip one question. Having graded the test, I have the following reflections:

- Students were generally successful on this test, and more successful at proof writing than other aspects of the test. The grade distribution was appropriate without a curve.
- With the test being so short, some students lost an excessive number of points by making one significant mistake. Problem 2 was the main culprit, with some students attempting to disprove a claim they should have proved. In the future, I will not make any one prove-or-disprove question worth so much of a test. But I otherwise think that the small number of questions was appropriate, as I wanted to test proof writing primarily.
- Some students found the notation in problem 3 confusing. I thought the ellipses would be friendlier than summation notation, but if I used this problem again I would use summation notation.
- The language in problem 5 "find a formula" should probably be more specific.
- It was hard to find a good strong induction problem for this test, and I basically didn't include one. This is less a comment about the test than the course itself, but I am motivated to spend more time early in the course introducing objects like graphs for which strong induction is a natural tool. The questions in the textbook that use strong induction are all a little contrived.

Test 2 Fall 2022

Name: ______________________________

This is an in-class test. The content is primarily from classes taught after the first test was published. You should complete these problems *on your own*. You may ask your instructor for clarifications. You may NOT use your textbook or notes or a calculator.

For problems which ask you to write proofs, you will be graded on the logic, writing, and accuracy of your proof. Your proofs may cite facts proven in class or on the homework. Some of these are given on a helpful page of references.

For other problems, you should still show any work needed to justify your answer in order to receive full credit. Your work should be organized and legible in order to be graded. Please turn in any scratch paper along with your test.

This test has five questions. You should choose four to complete, and indicate which question you are not completing by writing a big X in the box for its score. If you do not do this, I will choose an omitted question for you: my methodology will not be deliberately unfair, but it may not be to your advantage.

Problem	1	2	3	4	5	Total
Score						

You may find it helpful to refer to the following theorems.

Theorem 1. You got this.

The Quotient-Remainder Theorem. Given any integer n and positive integer d, there exist unique integers q and r such that

$$n = dq + r \quad \text{and} \quad 0 \le r < d.$$

Lemma (used in the Euclidean Algorithm). If a and b are any integers not both zero, and if q and r are any integers such that $a = bq + r$, then $\gcd(a, b) = \gcd(b, r)$.

Proposition 1. Regarding odd and even numbers.

1. The product of any even number and any integer is even.
2. Any finite product of odd numbers is odd.
3. Any finite sum of even numbers is even.
4. The sum of k odd numbers is odd if and only if k is odd.
5. The sum or difference of two numbers of opposite parity is odd.
6. The difference of two numbers of the same parity is even.

Proposition 2. Where $r_1, ..., r_k$ are rationals:

1. $\sum_{i=1}^{k} r_i$ is rational;
2. $\prod_{i=1}^{k} r_i$ is rational;
3. $r_1 - r_2$ is rational;
4. r_1/r_2 is rational.

Proposition 3. Suppose $x \equiv a \mod n$ and $y \equiv b \mod n$. Then:

1. $x + y \equiv a + b \mod n$;
2. $x - y \equiv a - b \mod n$;
3. $x \cdot y \equiv a \cdot b \mod n$.

Formula 1. $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$.

Formula 2. $\sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r}$.

The Handshake Lemma. The sum of the degrees of the vertices in a graph is twice the number of edges in that graph.

The Distinct-Roots Theorem.
Suppose a sequence $a_0, a_1, a_2, ...$ satisfies a recurrence relation $a_k = Aa_{k-1} + Ba_{k-2}$ for some real numbers A, B with $B \neq 0$ and every integer $k \geq 2$. If the characteristic equation $t^2 - At - B = 0$ has two distinct roots r and s, then $a_0, a_1, a_2, ...$ is given by the explicit formula $a_n = Cr^n + Ds^n$, where C and D are numbers whose values are determined by the values a_0 and a_1.

The Single-Root Theorem.
Suppose a sequence $a_0, a_1, a_2, ...$ satisfies a recurrence relation $a_k = Aa_{k-1} + Ba_{k-2}$ for some real numbers A, B with $B \neq 0$ and every integer $k \geq 2$. If the characteristic equation $t^2 - At - B = 0$ has a single root r, then $a_0, a_1, a_2, ...$ is given by the explicit formula $a_n = Cr^n + Dnr^n$, where C and D are numbers whose values are determined by the values a_0 and a_1.

1. Complete the following problems, which are unrelated to each other.
   (a) For each of the following degree sequences: draw a graph with that degree sequence if possible, otherwise state why it is impossible.
      i. 1,2,3,4,5
      ii. 3,3,3,3,3,3
   (b) What is the last digit of 3^{10000}? Show your work.
   (c) Assuming you are taking this test on a Thursday, what will be the weekday in 10000 days? Show your work.

   Hint. None

2. Prove or provide a counterexample to each of the following.
   (a) The sum of a rational number and irrational number is always irrational.
   (b) The sum of two irrational numbers is always irrational.

   Hint. At least one of these can be proven by contradiction.

3. Prove by induction that for any integer $n \geq 1$, the sum $1 + 2 + ... + 4n$ is even.

   Hint. The facts we have proven about odd and even integers can be cited to avoid applying the definition of even and odd several times. If you're really stumped, you can prove this without induction using one of the formulas for partial credit.

4. Prove by induction on n that for any integer $n \geq 0$,

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1.$$

   Hint. If you're really stumped, you can prove this without induction using one of the formulas for partial credit.

5. Find a formula for a_n, where $a_0, a_1, ...$ is the sequence satisfying the recurrence relation

$$a_{n+2} = 3a_{n+1} - 2a_n, \quad n \in \mathbb{N}$$

   and the initial conditions $a_0 = 1$ and $a_1 = 3$. Justify your answer.

   Hint. Your justification could be a sketch of a proof by induction, but you could also appeal to a big result from class. Either way, you don't need to write a formal proof.