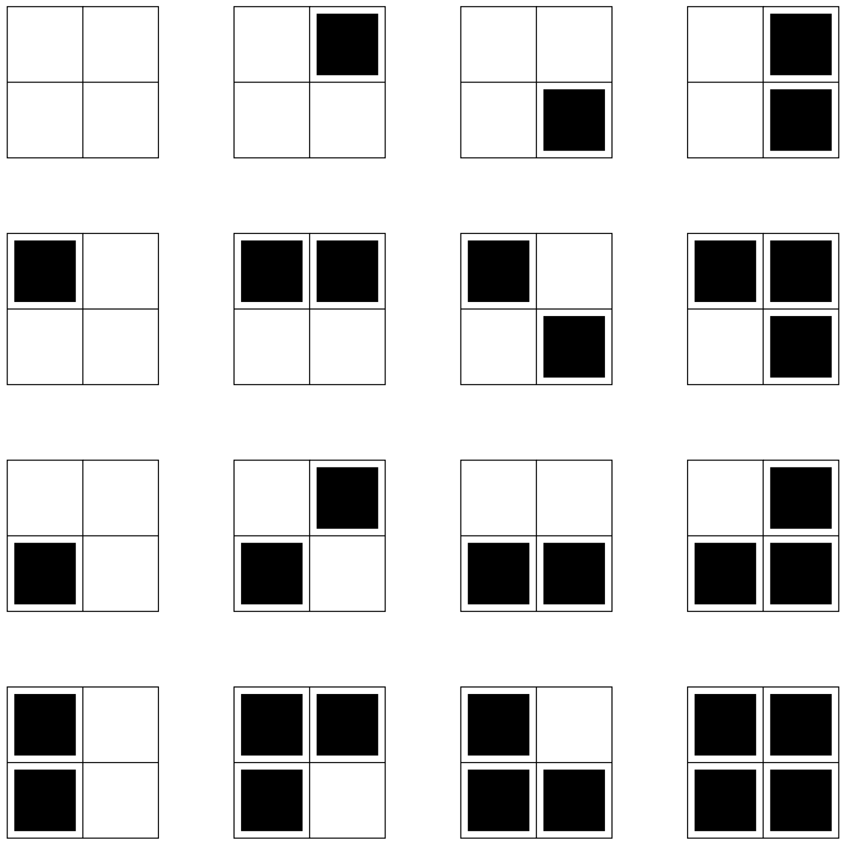
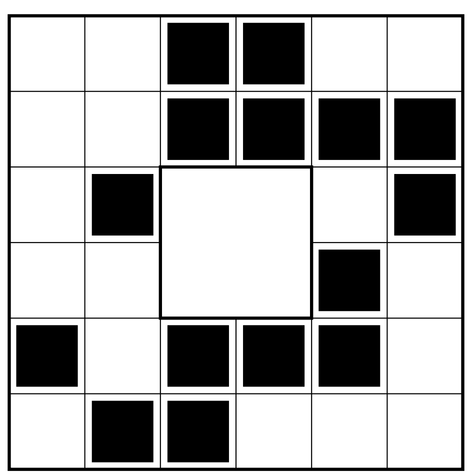
The 16 Squares Puzzle

There are 16 ways to color the cells of the fixed square tetromino (a 2×2 grid) either black or white.



Find the smallest (by number of cells) colored polyomino which contains all 16 of the configurations above. For example, the colored (doughnut-shaped) polyomino below contains each configuration exactly once, but a smaller solution is possible.



This problem can be broken down into steps:

- Identify a likely shape for the large colored polyomino
- Find a satisfactory coloring of that polyomino
- Show that no smaller polyomino can have a satisfactory coloring

Combinatorialists study the structure of objects by counting their variations, so after solving the puzzle we might ask:

- How many different solutions to this puzzle are there?
- How many solutions to the puzzle are there with n colors?
- Do the answers to these questions hint at a relationship between this problem and other problems we have seen before?

It turns out there are 800 solutions to the 16 Squares Puzzle, which can be generated quickly using a computer, but there are so many solutions to the analogous puzzle with 3 colors that the same approach may not be possible. To date, we don't have a general formula for the number of solutions to this kind of puzzle with n colors.

It's often helpful to study many variations of a problem. For example, one might ask what happens if we replace the square tetromino with another small polyomino, as well as changing the number of colors? What happens with other types of polyforms? For which variations of this problem can we find solutions, for which do simple solutions seem feasible, and for which do they seem inherently infeasible?